Large Scale Integration of Micro-Generation to Low Voltage Grids

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Methods for Allocation of Losses and Network Capital in Systems with MicroGrids

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Report

A Framework for Allocating Charges in Networks with Microgrids

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1. Introduction

One of the challenges in the operation of competitive electricity markets is the commercial imperative to assure open and non-discriminatory access to transmission and distribution networks to all participants in the energy market. In an environment where relationships between business entities are defined through commercial contracts, the issue of network open access is in essence a pricing problem. In this report, a framework for development of tariffs for distribution systems with distributed generation was developed based on the notion of the reference network and long run marginal cost pricing to provide cost reflectivity and stimulation for economic efficiency in the systems.

Historically, distribution network charging methodologies were derived based on the philosophy that the costs for providing adequate network capacity are mainly driven by demand customers in the peak demand condition. With no significant amount of generation embedded to distribution networks, this norm is valid. However, with the expected significant penetration of distributed generation (DG) and microgrids in the future, distribution network reinforcement could be driven not only by the requirement to accommodate demand growth but also by the demand to transport power from DG via distribution wires. Also, distributed generation may postpone the demand for network reinforcement and hence this generation should be adequately rewarded. This development opens up the question as to how to allocate network charges between demand and generation customers. Clearly, this question cannot be answered by the present charging methodologies that were not designed specifically to capture the contribution of DG to network costs.

The main objective of network charges to provide economic signals to network users regarding the costs they impose on the system, that are location and time-of-use specific. Firstly, this signals are important for location related decision of new DG and demand connections and their pattern of network use so that network reinforcement costs can be optimised. Without these signals, the existing and potential network customers will not be informed about the impact they make on network costs at particular location and pattern of use. Therefore, in order to achieve economic efficiency and to stimulate a balance between the network and generation costs, these economic signals are critically important.

Hence, the overall aim of the work conducted in this project is to develop a cost reflective charging methodology taking into account the contribution of demand and DG including micro sources on network capacity requirement. The methodology should include rewards to demand and generation customers for benefits that they may create in terms of providing system security, deferring the need for system investment, and etc.

The work was conducted as a contribution to MICROGRIDS project for Work Package G task G4[1] in investigating an appropriate charging methodology to recover

[1] Details of the task can be found in the MICROGRIDS project proposal [1].

the capital investment and operation cost of distribution networks characterised with the presence of distributed generation which in small scale represents microgrids and concurrently to provide economic signals for demand and generation customers. A consistent charging methodology that can be applied generally across various voltage levels, from Low Voltage (LV) microgrids up to Extra High Voltage (EHV) distribution systems where microgrids are connected to, is also desirable and becomes one of the objectives in this work.

In order to capture the temporal and spatial contribution of demand and DG on the network capacity requirement, a charging methodology based location specific and time-of-use specific entry and exit Distribution Use of System (DUoS) charges for every connection point on the system is proposed and described in this report. Entry and exit charges are applied for generation and demand customers respectively.

The report is organised as follows.

Chapter 2 describes two basic charging methodologies namely connection charges and DUoS charges, and various implementation of these two charging policies namely deep, shallower and shallow connection charges. As deep connection charges may incur significant barrier to DG, at present there is initiative from the electricity regulator in UK supported by DG developers to use shallower charges to stimulate more connection of DG in the distribution network. This initiative raises a question for cost reflective DUoS charges, which can take both positive and negative contribution of DG on the network costs. Chapter 2 also presents various critical aspects that need to be considered to derive cost reflective charging methodologies.

Chapter 3 describes the core philosophy of the proposed pricing methodology. The concept of time of use and location specific DUoS charges for exit (demand) and entry (generation) across various voltage levels of connection is illustrated and described using a simple example of generic distribution network architecture.

Chapter 4 provides mathematical models for the proposed pricing calculation described in the previous chapter. This chapter also provides with the formula to derive DUoS tariff for each node and to do revenue reconciliation.

Chapter 5 consists of the description of the case studies conducted to illustrate the implementation of the pricing methodology into microgrids. Impact of various level of DG penetration on the DUoS tariffs, revenue reconciliation process and a technique to pass charges from Public Electricity System (PES) to where the microgrids are connected are presented in this chapter.

Finally, the contents of this report are summarised in the last chapter.

2. Network charging methodologies

2.1. Basic charging policies

The capital expenditure and operation and maintenance costs of distribution networks are recovered through charging all network users consisting of generation and demand customers. In this section, two basic distribution network-charging methodologies are discussed.

1. Connection charges
2. Distribution Use of System Charges (DUoS)

These charges exclude energy services related charges such as top-up and stand-by charges, metering and data management charges.

Connection charges are non-periodic payments, generally one off up front payments or multiple payments imposed over an agreed period of time between Distribution System Operator (DSO) and the customers, who request a new connection, to recover an appropriate proportion of the capital cost of new infrastructure required for the new connection and the capitalised operating and maintenance costs. The proportion of costs that can be levied directly as connection charges is likely to be regulated. Only users who trigger the need of new infrastructure will be charged accordingly. Users who also use the infrastructure but do not trigger the need of reinforcement do not pay the charges. Connection charges are generally calculated case-by-case basis.

In contrast, DUoS charges are periodic payments and paid as long as users remain connecting to and using distribution networks. Network costs that need to be recovered through DUoS charges are allocated to all users depending on the adopted charging methodology. DUoS charges are generally based on the registered installed capacity (£/kW) or based on the energy utilisation (£/kWh) or a mixture between these two. DUoS charges are used generally to recover not only capital (Capex) expenditure but also operation (Opex) including maintenance expenditure. Furthermore, DUoS charges generally contain Transmission Use of System Charges that also need to be paid by distribution network customers.

Application of these two basic charging methodologies can be varied. Various charging policies[2] are listed as follows:

1. Deep connection charges
   The users who request a new connection pay all costs associated to the network reinforcement incurred by the connection through connection charges. This includes the cost of reinforced assets not only at the voltage of connection but also the costs incurred at upstream voltage networks. Furthermore, Operation and Maintenance (O&M) costs of the reinforced assets are capitalised, typically 10%-30% of the value of the assets and levied through connection charges.

2. Shallower connection charges

[2] The terms: deep, shallower and shallow connection charges are commonly used in England and Wales to identify various implementation of network charging methodologies

Part of the network reinforcement costs is recovered via connection charges and the remainder plus O&M costs are recovered via DUoS charges. The connection boundary specified by network companies and/or electricity regulators determines the network area where the cost is recovered through connection charges.

3. Shallow connection charges
   All network reinforcement costs plus O&M costs are paid via DUoS charges.

In England and Wales, a generation customer currently pays typically deep connection charges while a demand customer pays connection charges for the costs incurred by the connection up to one voltage level beyond the voltage of connection and pay DUoS charges for rest of the costs. Since deep connection charges may be very significant, it has been suggested that this may be preventing DG from entering the market. In the light of stimulating DG connection in the long term, generation customers should face shallower or shallow charges for connection to the distribution system. At present, DSOs and the UK electricity regulator, Ofgem, have intensive discussions and works in the area of preparing shallower cost reflective charging methodologies for DG that will be included in the next distribution pricing control scheme in April 2005.

In general, the use of shallow connection charges is desirable for users who request new connection because the network charges are paid in a long-term basis and spread across all customers who utilise the network. This eases the burden of customers to pay a relatively large amount of money up front. In addition, since network charges are distributed accordingly to all network users and not only to the users who trigger the need of reinforcement, the issues of "free ride" or dispute between DG developers and DSO over the allocation of network reinforcement costs can be avoided.

This report focuses on designing a DUoS charging methodology for distribution networks with distributed and micro generators.

2.2. Previous works in cost reflective electricity charges

Back in the early 1900's, Hopkinson, Arthur Wright, Gisbert Kapp, and L.R.Wallis [2] acknowledged various factors that need to be considered in order to derive cost reflective electricity charges. Although the works initially focused on pricing of electricity and not particularly in the DUoS charges, the fundamental concepts are also relevant for pricing of transmission or distribution networks. The development of pricing methodology has also been continued by many other researchers particularly in the area where Distributed Generation may have significant impact on the performance of distribution network investment. A problem of pricing distribution network with DG is relatively new and emerges recently due to the increase penetration of DG in distribution network.

The following points summarise various key aspects to derive cost reflective DUoS charges:

1. Critical loading. For security reason, network capacity is determined by the critical loading of the assets. Thermal overloading should be avoided to prevent insulation breakdown, malfunction or performance deterioration that may lead to

the loss of supply. However, higher capacity is associated likely with higher cost and hence customers who drive the critical loading should be penalised with higher charges. The application of this concept in electricity charges was introduced firstly in 1892 by Hopkinson. On the contrary, customers who alleviate and reduce the demand of system reinforcement should be rewarded. For example, in a distribution system without DG, the growth of demand can only contribute to the increase in network loading and the required network capacity. Therefore, the ability of local generation (DG) in displacing this capacity can be rewarded. On the contrary, a relative large penetration of DG may drive demand for network capacity and local demand customers could contribute in reducing the demand for capacity by absorbing power locally.

2. Time of use. Consistent with point 1,critical loading of various network assets is typically associated with demand peak load conditions. Hence, the use of system charges during this period should be higher than the charges in other periods. This concept of time of use tariff system was introduced firstly by Gisbert Kapp, the father of time of day pricing. This concept has been applied widely for metering electricity.

3. Location. Customers who use more assets as media for transporting power should be charged more than customers who use less. Therefore, the charges should reflect on the contribution of customers depending on the location and other parameters such as network topology and parameters, to the critical loading of network assets.

4. Utilisation. It is arguable that at certain extent the customers who utilise highly the network should be charged more than the customers who utilise less. This can be indicated by the amount of energy consumed or injected to the network for a particular period of time. Another alternative of having charges based on the kilowatt of maximum power consumption is to have charges based on the kilowatt-hour.

2.3. Issues for determining the suitable charging methodology

In order to determine the suitable charging methodology, there are some issues to be considered.

1. Issues for network companies
    - The level of certainty for recovering fully the investment cost and obtaining an appropriate level of profits. Hence the companies should justify the risk of financial loss due to adopting a particular DUoS charging methodology. For example, a relatively accurate forecast of the total or individual energy consumption is required for implementing utilisation charges (£/kWh). The company carries a financial risk of not being able to generate the expected revenue if the actual energy consumption is lower than the forecast. In this situation, it may be a desired option to choose a methodology, which could relatively give firm expected revenue.

- Cash flows. In order to finance smoothly activities in distribution companies, good management of cash flows is essential. Immediate payment is desirable since it gives flexibility to manage the cash internally and reduce the risk.

- Cost of billing systems. The costs comprise cost of infrastructure (metering, communication, computer), cost of managing and operating the billing system, publishing tariffs, metering the utilisation from each customers, sending the bills and receiving the payment, dealing with customers complaints regarding the bills, and etc. Since a complex charging methodology will likely be expensive, a relatively simple mechanism could be more attractive for implementation.

2. Issues for network customers
   - Cost reflectivity
   - Transparency
   - Audit-ability
   - Affordability

3. Issues for electricity regulators
   - Sustainable of distribution business. The monopoly nature of network companies forces the regulators to determine the allowable revenue, which guarantees the recovery of the investment plus an acceptable level of profits in the distribution businesses. Adequate investment recovery is essential for this business to be economically feasible. Adequate incentive also needs to be provided to stimulate better management of distribution businesses.
   - Economic efficiency. Network tariffs can be used as economic signals to the network developers and users to design, invest and utilise the network optimally.
   - Stimulation to the development of renewable and distributed generation. It is important that the charges are affordable to the new connection and the existing DG to attract new investments. The pricing methodology, which can reward the positive contribution from DG to the network expenditure, will be definitely attractive for DG.

2.4. Distribution Reinforcement Model charging methodology

In England and Wales, a charging methodology known as the Distribution Reinforcement Model (DRM) is currently used to evaluate DUoS charges especially for demand customers. This model is employed to evaluate the long run marginal cost of expanding, maintaining, and operating the distribution system. This is achieved by calculating the network cost of adding a 500 MW load on the system maximum demand.

These costs are then allocated across voltage levels and customer groups such that the resulting DUoS charges are somewhat cost reflective. This is achieved by identifying the contribution of each customer group to the long-term distribution system cost. The resulting tariff takes the form of maximum demand and/or unit related charges. Maximum demand charges are used for levels of the system close to customers. This is based on the argument that customers will occupy fully the capacity of the local network to which they are connected. These charges are usually expressed in terms of £/kVA/month. On the other hand, unit based charges in £/kWh reflects the impact on the network cost further up the system. This approach is supported by the argument that the customer individual maximum demand is less likely to coincide with the system maximum demand.

Although the tariffs are designed to be cost reflective, a number of simplifying compromises are made in the implementation phase. For example, by having policies that urban and rural customers pay the same charges, although the costs of supplying rural customers are generally higher than those in urban areas. At present, this cross-subsidy might be considered socially desirable.

It is important to bear in mind that distribution use-of-system tariffs have been developed for customers who take power from the network rather than for customers who inject power into the network. In the context of the objective to facilitate the developments in distributed generation and microgrids, it becomes important to develop a pricing regime that will recognise the impact that distributed generation makes on network costs. One of the key issues is the economic efficiency of tariffs and their ability to reflect cost streams imposed by the users, particularly distributed generation.

The impact of distributed generation on distribution networks (in terms of costs and benefits) is site specific, it may vary in time, will depend on the availability of the primary sources (important for some forms of renewable generation), size and operational regime of the plant, proximity of the load, as well as the layout and electrical characteristics of the local network, etc. It is not, therefore, surprising that the relatively simplistic DRM tariff structure, with network charges being averaged across customer groups and various parts of the network, cannot reflect the cost impact of distributed generation on the distribution network.

It should be noted that DRM tariffs have no real ability to capture the impact of multi-directional flows (caused by the presence of distributed generation) and cannot deal with the temporal and spatial variations of cost streams. The developed model should therefore be able to take into account changes in directions of power flows driven by distributed generation.

A summary overview of costing and pricing of distribution networks together with a description of the DRM are presented in reference [3]. Furthermore, a discussion on various connection and DUoS charging options including a description of the concept of entry-exit charging methodology are presented in reference [4].

3. Framework for development of tariffs for distribution networks with distributed generation

3.1. Scope

This section presents the core features of the newly proposed cost reflective method for pricing of distribution networks with distributed generation. Details of the methodology and processes for its practical implementation are explained in this chapter. This pricing framework is relevant and applicable for microgrids. It is important to note that the proposed method only considers network related costs and the cost of micro sources, energy storages and load controllers are excluded from the methodology.

Location specific, time of use entry-exit DUoS charging methodology has been proposed taking into account multi-directional power flows driven by the presence of distributed generation including micro sources, and characteristic of the existing distribution system at all voltage levels including network design practices together with corresponding load and distributed generation characteristics that are relevant for network reinforcement.

The scope of this report focuses on addressing the allocation of thermal capacity driven costs, which are the major component of network costs. Allocation of network costs driven by fault level contribution and losses are also important and need to be addressed in the future.

3.2. Desirable attributes of network pricing methodology

The framework for pricing of distribution networks including distributed generation should have the following attributes:

1. Revenue requirements; developed network tariffs should yield sufficient amount of revenue to allow efficient operation and development of distribution networks.

2. Economic efficiency; network costs should be allocated so as to reflect the true cost that each group of users (or individual user, if large enough) imposes on the distribution network, i.e. it should avoid cross subsidies between different users and between different times of use.

3. Future investment signalling: ability to send clear cost messages regarding the location of new generation facilities and loads. Furthermore, pricing regime should signal the need for and location of new distribution network investments, i.e., encourage efficient network investment and discourage over-investment

4. The network cost allocation method should be equitable, auditable and consistent.

5. Must be practical to implement; any proposed network cost allocation method should balance the economic efficiency of tariffs and their complexity and social objectives. Furthermore, from a practical standpoint the allocation method should

be easy to understand and implement, and it is desirable to base it on actual metered data.

3.3. Philosophy of the proposed pricing approach

The overall philosophy of the proposed approach is based on the concept of reference network. The reference network has the same topology and loading conditions as the real system, while the capacity of various items of the plant is optimal. Since the loading of the plant is the main design parameter used in this study, the reference capacity (rating) will be equal to the maximum loading of the plant.

In order to determine the reference rating of individual plant items two loading conditions are considered: one that corresponds to the maximum demand and minimum generation condition and one that corresponds to the minimum demand and maximum generation condition. The use of these two conditions reflects the practice in designing the capacity in distribution network.

Network assets are then classified into three classes: demand dominated, generation dominated or balance. If the critical loading of a plant occurs during maximum demand condition, the plant is called a demand dominated (DD) plant. On the other hand, if the critical loading occurs during maximum generation condition, the plant is called a generation dominated (GD) plant. If both conditions influence the critical loading, the plant is called a balanced (BB) plant.

It is important to note that the methodology can cope with any number of loading conditions. However, increasing the number of system conditions considered in the formulation will increase the complexity of managing the tariffs. Bearing in mind that the number of sets of tariffs is equal to the number of considered loading conditions. In this report, two loading conditions are assumed to be adequate to demonstrate the functionality of the proposed methodology.

Since generators and demand have opposite effects on the plant loading during the period of critical loading of the plant, both positive and negative charges will be present. Users that tend to reduce the loading of the plant during this period will be rewarded for the use of the network. In case of a demand dominated plant downstream generators get paid, while downstream demand pays for the use of the plant. Similarly, if the plant is generation dominated, demand gets paid, while the generators pay for the use of the plant. On the other hand, charges outside of the period of maximum plant loading are zero since the incremental change in plant loading does not require plant reinforcement and hence does not impose any capacity related cost.

3.4. Illustration

An illustrative example is developed in the following text to facilitate the description of the concept of the proposed pricing models and its main characteristics.

3.4.1. The system

Consider a part of a radial distribution network[3] supplied from a 132kV/33kV substation with two transformers (Figure 3-1). Two 33kV out-going circuits are shown, while the rest of the 33kV network is represented by a lumped load of 50MW maximum demand, connected at the 33kV busbar. The two 33kV circuits supply a 33kV/11kV substation with two 33kV/11kV transformers. At the 33kV busbar of the substation a 15 MW CHP plant is connected.

From this substation, two 11kV feeders are explicitly represented, while the rest of the 11kV network is represented by a lumped load of 10MW maximum demand.

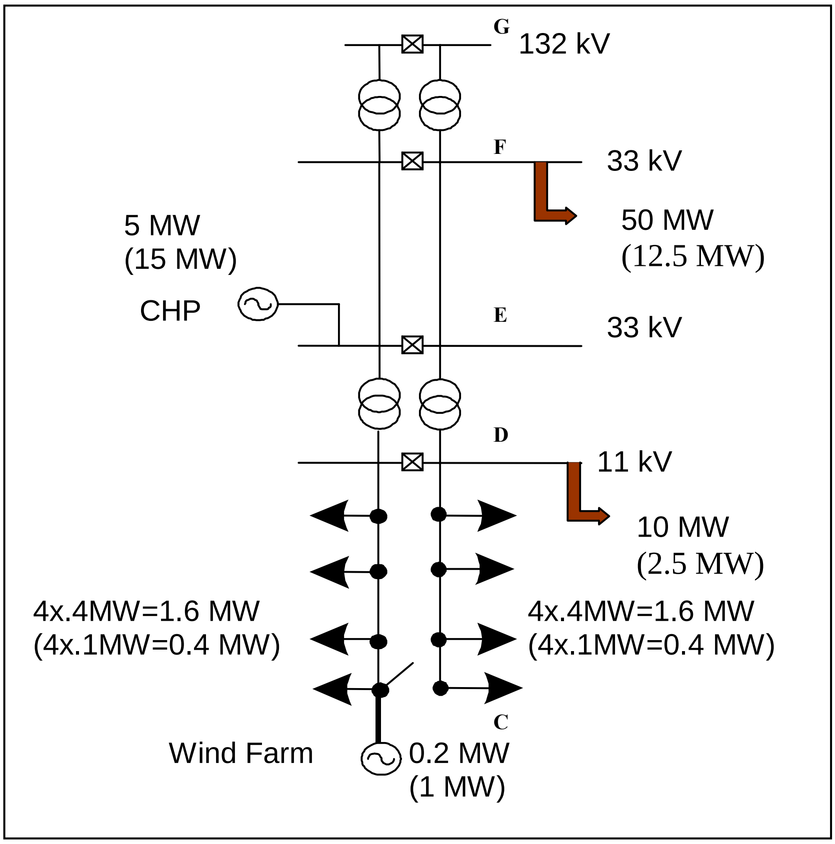

Figure 3-1: Example system

Each of the 11kV circuits supplies four 11kV/0.4kV transformers with a maximum demand of 400kW[4]. The LV systems can consist of microgrids and other ordinary LV networks. A Wind Farm of 1MW is connected to the circuit to the left.

3.4.2. The loading conditions

In practice, a distribution network is designed to cope with the expected maximum loading condition, which likely occurs at a time of maximum demand with minimum local generation. With DG, another extreme condition needs to be considered, i.e. the

[3] Typical structure of distribution networks in UK
[4] In this example, the effect of diversity in demand is not taken into consideration.

condition where DG produces maximum output and demand is minimum. Therefore, in this example, these two loading conditions are considered. The system loading is shown in Figure 3-1. The first number (without brackets) is the loading during maximum demand regime and the second number (in brackets) is the loading during maximum generation regime.

Since the design of the distribution networks should take into account security, information regarding the contribution of distributed generation to network capacity (security) is required. For the sake of this illustrative example, effective contribution of 5MW is allocated to CHP, and 200kW to the Wind Farm. In the context of DUoS pricing, it could be interpreted simply that the CHP and Wind Farm are capable of replacing a distribution circuit of the capacity of 5MW and 200kW respectively. This means that the firm capacities of these generators in the minimum generation regime are 5 MW and 200 kW respectively.

Minimum demand is assumed to be 25% of the maximum demand. This information is important since the condition of maximum generation and minimum demand may be critical for design/reinforcement of some of the items of plant in the network. It is important to note that the loading condition taken for the pricing calculation should be consistent with the loading scenarios taken in the network design process.

3.4.3. Reference network and charges

The flows in both loading conditions can be obtained by simple inspection. Critical flows and flows for two loading periods are summarised in Figure 3-2. The arrow shows the direction of the flows. The critical loading of plants can be determined by the largest power flows between two loading periods.

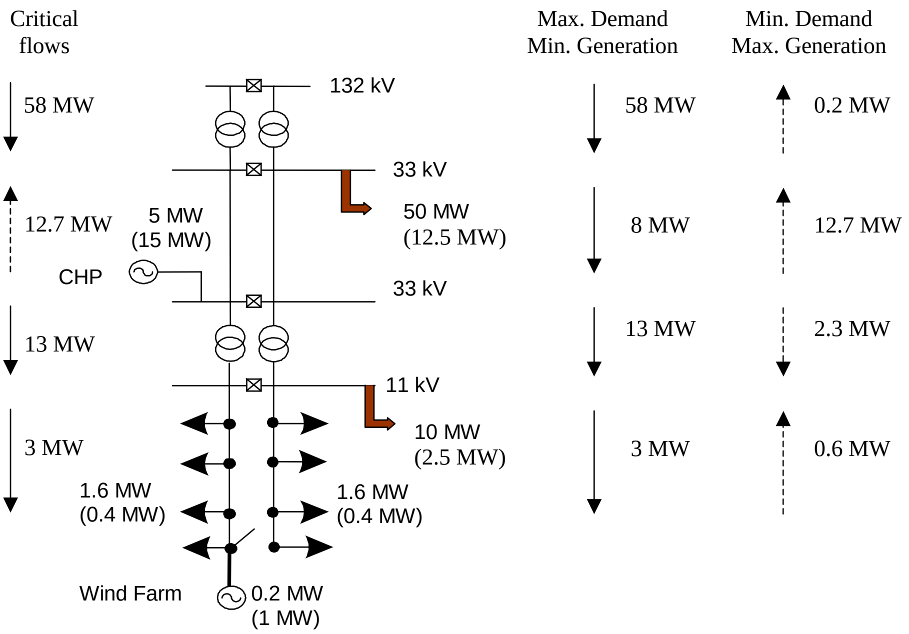

Figure 3-2 Critical flows in the system

It can be observed that these flows are not simultaneous: critical loading for the 11kV feeder, 33kV/11kV and 132kV/33kV transformers is driven by maximum demand, while critical loading of the 33kV circuits is driven by maximum generation, and these occur at different periods (time of use). Furthermore, it can be observed that the loading on 11kV network, 33kV/11kV and 132kV/33kV transformers is Demand Dominated (DD), while the loading of the 33kV network is Generation Dominated (GD).

Note that the critical flows determine the reference (optimal) ratings of the corresponding plant. The reference rating of 11 kV and 33 kV circuits are 2x3 MW and 2x12.7 MW respectively. Due to topology of 11 kV circuits, one feeder must cope with all 11 kV loads when one of the 11 kV feeders loses supply from the 33kV/11kV substation and the Normally Open point is closed. The optimal rating of the 132kV/33kV substation is 2x58MW. These reference ratings of the individual network components (transformers and lines at various voltage levels) can be compared with the plant ratings of the existing network.

3.4.4. Balancing Point

In the entry-exit pricing model, it is important to introduce an energy balancing point at which all electricity is considered to be exported and imported. This is presented in Figure 3-3.

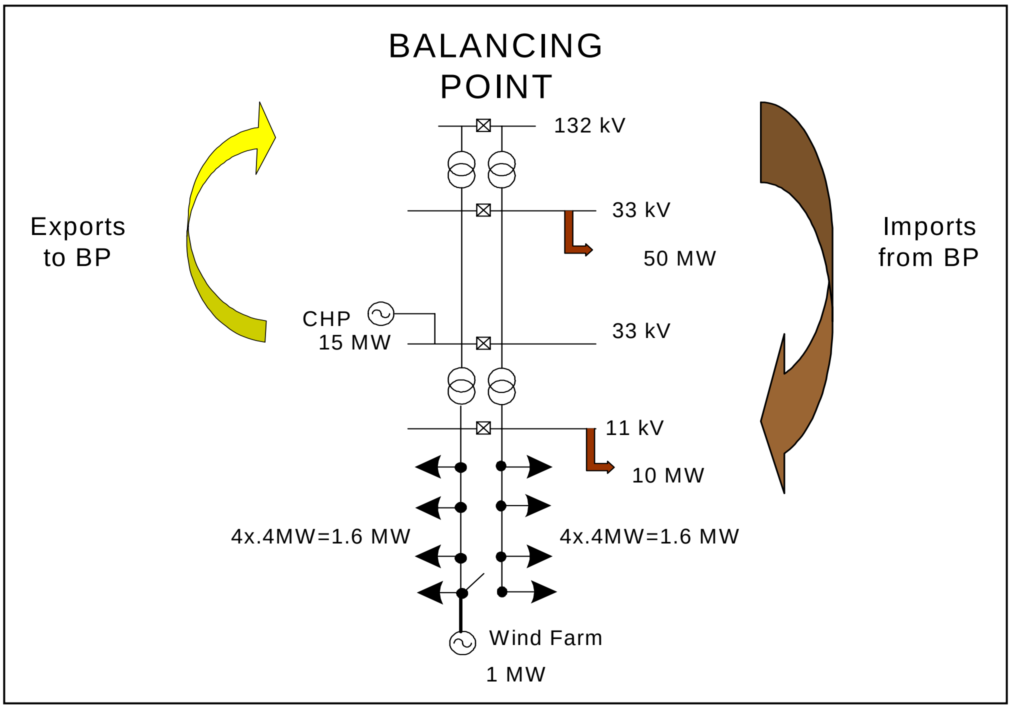

Figure 3-3 ***Balancing point***

The balancing point chosen (as shown in Figure 3-3) represents the boundary between the transmission and distribution networks. All distributed generators are deemed to be exporting electricity to this point (light shaded arrow in Figure 3-3) and all demand is deemed to be importing electricity from the same point (dark shaded arrow in Figure 3-3).

The balancing point also provides reference charges, which are not affected by the network costs downstream.

3.4.5. Polarity of exit and entry DUoS charges

Given the direction of the critical flows and knowing the direction of demand and generation driven flows (Figure 3-2), the polarity of exit and entry DUoS charges can now be determined. If the direction of the critical flow in the plant coincides with the direction of the flow imposed by a particular network user, this user will be charged for the use of the plant. On the other hand, if the direction of the critical flow is opposite to the flow created by a particular user, this user will get paid for the use of the plant. This is presented in Table 3-1 and Table 3-2, for demand (exit) and generation (entry) respectively.

Table 3-1 Polarity of DUoS exit charges at various voltage levels

Load	Plant			
Connection	11 kV	33/11 kV	33 kV	132/33 kV
11 kV	Pay	Pay	Get paid	Pay
33/11 kV	0	Pay	Get paid	Pay
33 kV	0	0	0	Pay

Table 3-2 Polarity of DUoS entry charges for the wind farm and the CHP plant

Generation	Plant			
Connection	11 kV	33/11 kV	33 kV	132/33 kV
11 kV	Get paid	Get paid	Pay	Get paid
33 kV	0	0	Pay	Get paid

3.4.6. Basis for evaluation of charges

Observe that positive and negative charges for a particular user are imposed during different periods. This is because the basis for the evaluation of positive charges is different to one for the evaluation of negative charges. For example, a positive incremental change in load of the demand connected to an 11kV feeder, during the maximum demand periods, will increase the loading on the 11 kV feeder, 33kV/11kV and 132kV/33 transformers. Therefore, charges for the use of the plant concerned (DD) will be based on maximum demand of 3.2MW.

Regarding the use of the GD 33kV circuit, the relevant critical period is determined by the coincidence of maximum generation and minimum demand. Hence, demand connected at 11kV will be rewarded for the use of this 33kV circuit, based on the load during minimum demand of 0.8MW. This is illustrated in Table 3-3.

Table 3-3 Basis for DUoS exit charges at various voltage levels

Load	Plant			
Connection	11 kV	33/11 kV	33 kV	132/33 kV
11 kV	3.2 MW	3.2 MW	0.8 MW	3.2 MW

33/11 kV	0 MW	10.0 MW	2.5 MW	10.0 MW
33 kV	0 MW	0 MW	0 MW	50.0 MW

Consider now charges for the wind farm. The wind farm will be rewarded for the use of the 11kV network and 33kV/11kV and 132kV/33kV transformers and will be charged for the use of the 33kV circuits. Again, the basis for the evaluation of positive charges is different to that of the evaluation of negative charges. The rewards for using the plant concerned will be based on the generator effective contribution (0.2MW). On the other hand, the charges for the use of the 33kV circuit will be based on maximum output (1MW). This is illustrated in Table 3-4.

Table 3-4 Basis for DUoS entry charges at various voltage levels

Generation	Plant			
Connection	11 kV	33/11 kV	33 kV	132/33 kV
11 kV	0.2 MW	0.2 MW	1.0 MW	0.2 MW
33 kV	0 MW	0 MW	20.0 MW	10.0 MW

It can also be observed that the proposed pricing approach captures correctly the interactions between network users and their composite impact on network use, and hence cost. The charges imposed to an individual user will depend on the critical loading of the plant upstream from the point of connection concerned. For example, the presence of CHP plant impacts on the charges for the wind farm and demand connected to the 11kV network (at and beyond 33kV), while the presence of the CHP plant does not impact on the charges at 11kV[5] (see Figure 3-3.)

Given the cost of individual plant expressed in £/kW/year, charges for individual network users can be evaluated. The exit and entry charges that are calculated on this basis will be cost reflective. This is achieved by the design of charges that are location and time-of-use specific. Note that the revenue accrued by imposing such charges will recover the cost of optimal plant capacity, driven by the critical flows, as presented in Figure 3-2.

3.4.7. Calculation and allocation of network charges

In order to evaluate network charges for individual users, per unit annuitised capacity costs (£/kW/year) are allocated to each plant in the network. For illustrative purposes, the estimate annuitised yardstick capacity costs of the 132kV circuits, 132kV/33kV transformers, 33kV circuits and 33kV/11kV transformers for typical rural and urban network in UK and the yardstick capacity costs used in the DRM model are presented in Table 3-5.

As shown in Table 3-5, the cost of supplying a rural customer is greater than the cost of supplying an urban customer. The largest differences in the cost between urban and rural networks are highlighted. The length of the rural 132kV network is significantly larger than that of the urban one, hence a larger cost (although the urban

[5] With the balancing point being at 132kV level, the CHP plant located at the 33kV is deemed not to be using downstream plant.

132kV network is typically underground). Furthermore, the marginal cost of a 50kVA 11kV/0.4kV transformer is significantly higher than one of 500kVA.

Table 3-5 ***Yardstick network cost for generic urban and rural networks***

	RURAL £/kW	URBAN £/kW	DRM £/kW
132 kV Circuits	23.2	12.1	13.5
132kV / 33kV Substations	5.9	5.2	6.3
33 kV Circuits	4.6	6.7	7.3
33kV / 11kV Substations	5.3	4.3	6.3
11 kV Circuits	12.8	11.0	10.5
11 kV / LV Substations	15.1	5.7	12.2
LV Circuits	10.8	9.4	10.4
	77.7	**54.5**	**66.7**

Given a generic entry-exit pricing model of the distribution network relevant to a particular network user, its DUoS charges can be easily evaluated by identifying the character of the upstream assets, in the sense of being demand or generation dominated. For example, a demand customer will pay for the use of all upstream assets that are demand dominated and get rewarded for the use of all upstream assets that are generation dominated. Of course, the basis for the evaluation of the corresponding DUoS charges and rewards will be different, as demonstrated by the previous example. The opposite is valid for generation customers.

The developed entry-exit pricing model is applied to the test system. The system is presented again in Figure 3-4 (see Figure 3-2), with all critical loadings highlighted. Next to the network model in Figure 3-4, the yardstick costs (urban) of individual plant items are presented.

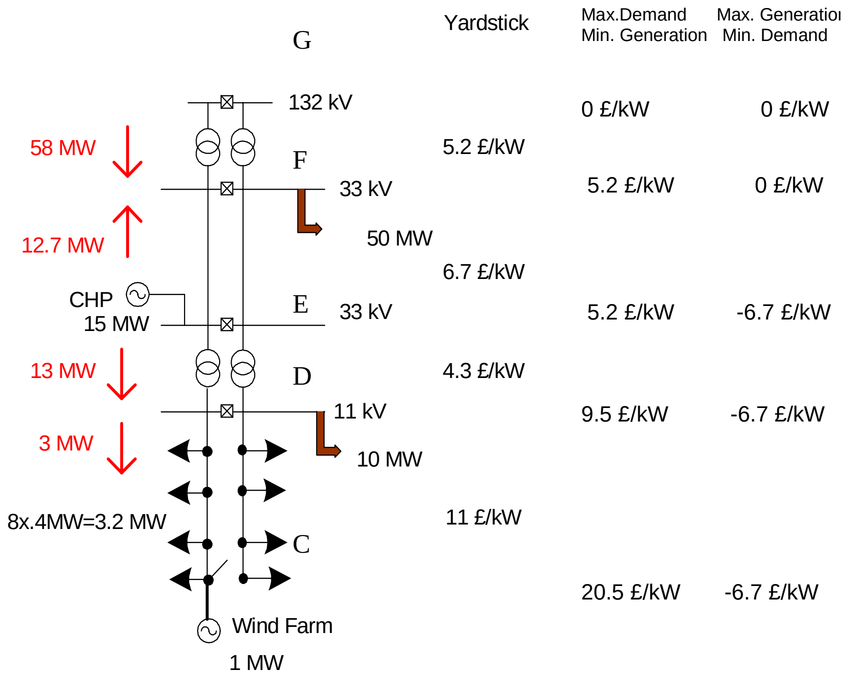

Figure 3-4 Evaluation of DUoS exit charges for the example given in Figure 3-4.

In Figure 3-4, DUoS exit charges for demand customers connected at various points in the network for the considered two loading periods are also listed. The polarity of charges is adopted to be positive for downstream and negative for upstream power flows respectively. DUoS entry charges for generation customers have the same magnitude but opposite sign with DUoS exit charges for the corresponding point of connection and time of use.

Since in this example the notional balancing point is the 132kV busbar of the 132kV/33kV, the DUoS charges for demand and generation customers connected at this point are pre-specified. In this case, the charges at balancing point are set to zero. However, it is also possible to have non-zero charges at balancing point. This feature will be useful especially if it is required to pass charges from transmission to distribution network users.

Consider now the 132kV/33kV transformer. This is demand-dominated plant since the direction of the power flow is downstream. Hence, all downstream demand and generation customers pay and are paid £ 5.2/kW/year respectively for the use of this particular plant during maximum-demand conditions while charges are zero during the minimum demand period.

The next plant to be considered is the 33kV circuit. This is a generation-dominated plant since the direction of the critical power flow is upstream. Hence, all downstream generation and demand customers pay and are paid £ 6.7/kW/year respectively for the use of this plant during maximum generation condition while zero is charged during the maximum demand period.

Hence, as shown in Figure 3-4, the total DUoS entry charges for generation customers connected to 33kV busbar of the 33kV/11kV transformer is - £ 5.2 /kW/year applied during the maximum demand period (- £ 5.2/kW/year for the use of the 132kV/33kV transformer and £ 0/kW/year for the use of the 33kV circuit) and DUoS of £ 6.7/kW/year during minimum demand period (£ 0/kW/year for the use of the 132kV/33kV transformer and £ 6.7/kW/year for the use of the 33kV circuit use). DUoS exit charges for demand customers are equal in magnitude but have reverse polarity.

The 33kV/11kV transformer is demand-dominated plant since the direction of the critical power flow is downstream. Hence, all downstream demand customers are charged and all downstream generation customers are paid £ 4.3/kW/year for the use of this particular plant during maximum demand conditions while the charges are zero during the minimum demand period.

Hence the total charge for the generation connected to the 11kV busbar of the 33kV/11kV transformer is -£ 9.5/kW/year during peak demand and minimum generation period (- £ 5.2/kW/year for the use of the 132kV/33kV transformer, £ 0/kW/year for the use of the 33kV circuit and -£ 4.3/kW/year for the use of the 33kV/11kV transformer) and DUoS entry charges of £ 6.7/kW/year during minimum demand period (£ 0/kW/year for the use of the 132kV/33kV transformer, £ 6.7/kW/year for the use of the 33kV circuit and £ 0/kW/year for the use of the 33kV/11kV transformer).

Finally, the 11kV feeder is demand-dominated plant since the direction of the critical power flow is downstream. Hence, all downstream generation customers are paid £ 11/kW/year for the use of this particular plant during maximum demand conditions while the charge is zero during the minimum demand period. Therefore, the total charge for generation customers connected to the 11kV circuit is -£ 20.5/kW/year during on-peak period (- £5.2/kW/year for the use of the 132kV/33kV transformer, £ 0/kW/year for the use of the 33kV circuit, -£ 4.3/kW/year for the use of the 33kV/11kV transformer and -£ 11/kW/year for the use of the 11kV circuit) and DUoS entry charges of £ 6.7/kW/year during minimum demand period (£ 0/kW/year for the use of the 132kV/33kV transformer, £ 6.7/kW/year for the use of the 33kV circuit, £ 0/kW/year for the use of the 33kV/11kV and zero for transformer and for the use of the 11kV circuit). All of this information is presented in Figure 3-4.

3.4.8. Cash flows

The DUoS charges (assuming positive polarity for demand customers) and revenues collected from various users during peak demand and off-peak demand conditions are given in Table 3-6 and Table 3-7 respectively[6]. The connection point G corresponds to the balancing point. Note that 58 MW is imported under peak demand conditions, while 0.2 MW is exported under minimum demand condition from the Grid Supply Point

Table 3-6 On peak demand DUoS prices and revenues from demand and generation customers

Connection point	Price £/kW	Demand MW	Generation MW	R Demand £	R Gen £	Total £
G	0	0	58	0	0	0
F	5.2	50	0	260000	0	260000
E	5.2	0	5	0	-26000	-26000
D	9.5	10	0	95000	0	95000
C	20.5	3.2	0.2	65600	-4100	61500
				420600	-30100	390500

Table 3-7 Off peak demand (peak generation) DUoS prices and revenues from demand and generation customers

Connection point	Price £/kW	Demand MW	Generation MW	R Demand £	R Gen £	Total £
G	0	0	-0.2	0	0	0
F	0	12.5	0	0	0	0
E	-6.7	0	15	0	100500	100500
D	-6.7	2.5	0	-16750	0	-16750
C	-6.7	0.8	1	-5360	6700	1340
				-22110	107200	85090

Note that during the peak load condition, the annual revenue is collected for all demand-dominated assets, while for the generation-dominated plant revenue is

[6] The first column corresponds to the balancing point. Note that 58MW is imported under on –peak conditions from the Grid Supply Point, while 0.2MW is exported under minimum demand condition from the distribution network to the transmission system.

recovered during peak generation periods. The costs of the individual plant items for the reference rating are given in Table 3-8.

Observe that, in this particular case, the total annual revenue received for the demand-dominated plant is £ 390,500/year, as shown in Table 3-6. (This is exactly equal to the total costs of the individual plant items as shown in Table 3-8, i.e. £390,500 = £301,600+55,900+33,000.)

On the other hand, the total annual revenue received from DUoS charges during the off peak demand period is £85,090, as shown in Table 3-7. This is exactly equal to the total cost of generation-dominated plants, as shown in Table 3-8.

Table 3-8 ***Annuitised cost of individual plant items***

Plant	Yardtick £/kW	Max flow MW	Cost £
Transf 132kV/33kV	5.2	58	301600
Circuit 33 kV	6.7	12.7	85090
Transf 33kV/11kV	4.3	13	55900
Circuit 11 kV	11	3	33000
			475590

Clearly, the reference exit-entry charges will recover the cost of plant of reference rating.

The on and off-peak demand DUoS related expenditure of individual users is presented in Table 3-9. It is evident that the total annual DUoS revenue equals the total annuitised cost of the reference network.

Table 3-9 ***Annual DUoS charges for individual network users***

User	On Peak Charge £	Off peak charge £	Total Charge £
Demand connected at F	260,000	0	260,000
Generator connected at E	-26,000	100,500	74,500
Demand connected at D	95,000	-16,750	78,250
Demand connected at C	65,600	-5,360	60,240
Generator connected at C	-4,100	6,700	2,600
			475,590

Note that charges capture the interactions between voltage levels.

4. Data and mathematical formulation for the proposed pricing methodology

This section focuses on the modelling and implementation of the proposed concept described in the previous section. The methodology consists of the following three basic steps:

1. Determination of thermal driven optimal network capacity to create reference network
2. Allocation of resultant reference network costs to users of the network
3. Revenue reconciliation

Before going to the detail description of each step, the required data for this methodology is described in the following section.

4.1. Data requirements

In general the basic data required to perform the calculation is listed below:
1. Generation profile data (location and amount of generation)
2. Demand profile data (location and amount of load)
3. Network data (topology, impedances, length, existing maximum capacities)
4. Network contingency list depending the network operating policies for each voltage level
5. Unit investment cost data (circuits, substations, transformers)

Data for network, demand, generation and contingency are given for each period. In this report, two scenarios namely maximum demand minimum generation and maximum generation minimum demand scenarios are used. The use of these two loading scenarios conforms to the network design practice.

Data on existing capacities of circuits is used to compare the calculated optimal capacities with real network capacities and to calculate the real network costs, which are required for revenue reconciliation.

Unit investment cost data is used later in the computation of the optimal investment cost and hence use of system charges. In the proposed approach the network capacity investment costs are assumed to vary linearly with capacity. Operating and maintenance costs could also be allocated to the network with respect to voltage level and type of plant. In practice, the increment of network costs is likely to be non linear and discrete; however, the use of linear marginal cost has been widely used to provide economic signals to users (and not necessarily the exact network increment cost). The problem can then also be simplified and it reduces the sophistication of computing the charges.

4.2. Determination of optimal network capacity

The central issue in network design for pricing is to determine the optimal capacity of individual network plant items and hence the optimal investment costs (and revenue) for the network owner/operator. In this approach the optimal network capacity for secure transport of electricity is calculated as the minimum capacity required for transporting power from generation sources to load centres assuming loads and generation dispatch are known upfront. A multi-period load model is adopted in order to capture temporal variations in demand and generation. While the load model in this work is limited to two periods representing maximum demand minimum generation and maximum generation minimum demand conditions, the algorithms that are developed can cope with any number of periods. However, increase in the number of demand periods will lead to increase number of time specific tariffs and therefore it leads to more complex tariff structure.

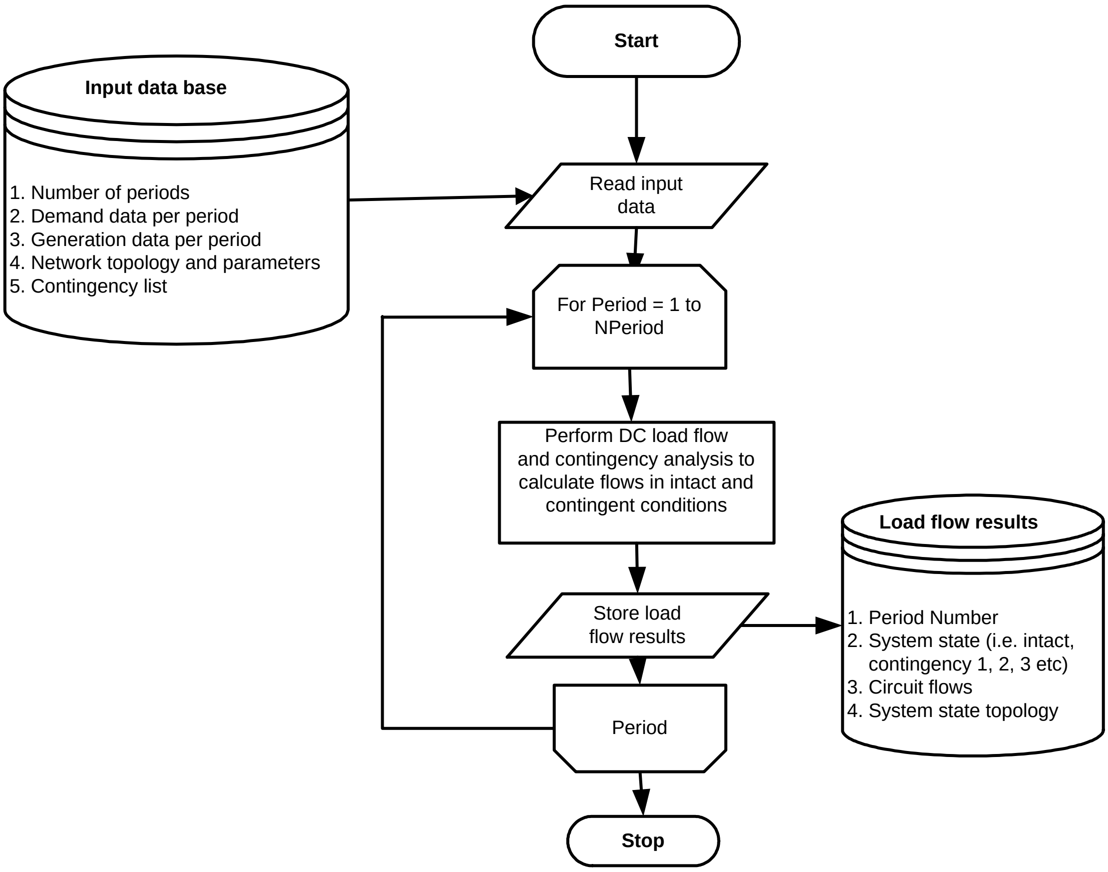

Figure 4-1 Steps in determination of critical flow

Because the load and generation for the peak demand and peak generation periods are known a-priori, the problem of determining the security constrained optimal network capacity of the network components can be decomposed into a series of dependent load flow problems representing the intact system and contingency states for each demand period. Critical circuit flows are then found by simply searching for the maximum circuit flows from the set of flows obtained from the series of load flows representing the various system states in each demand period. This process for determining the optimal circuit capacities can be explained with the aid of the flow chart shown in Figure 4-1. The process starts by reading in the required data (the data requirements are summarised in Chapter 4.1). Then the load flows for period one are performed starting with intact system load flow followed by load flows for each credible contingent state. The circuit flows are stored after each load flow. When load flows for all the periods and contingencies are completed and results stored, a routine to calculate the price is called. The flow chart for this routine is shown in Figure 4-2.

The starting point for this routine is to determine the optimal capacity of the network circuits. As explained already above, the critical circuit flows are found by searching for the maximum circuit flows from the set of flows obtained from the stored circuit flows obtained from the series of load flows for the various system states in each demand period.

4.2.1. Use of DC load flow algorithms

As stated above critical flows (or optimal circuit flows) are determined by performing a series of load flow computations. Load flow studies can be undertaken using an AC or DC model of the network. The choice of methodology depends on the indented use of the results. In pricing studies where network capacities are important and voltage variations as well as reactive power flows are not generally critical, it is usual practice to adopt the DC load flow.

Using realistic data for 132kV, 33kV, 11 kV and 0.4 kV networks, we have carried out both AC and DC load flow studies. A comparison of the results clearly demonstrated that the application of DC load flow was adequate. Since distribution networks are typically radial, power flows are not significantly influenced by network impedances as in the case of meshed networks. A typical error within the range of 1% - 5% is observed. This small error will not impact significantly the results of the pricing methodology.

The following assumptions are made in relation to the DC load flow implementation in this work:

1. Voltage drops are negligible, hence all voltage magnitudes are equal to 1.0 p.u.
2. Losses are ignored

The main advantages of using DC load flow include the following:

1. Load flow is linear and therefore does not require iterative techniques considerably speeding up execution times of the algorithm. This is clearly a benefit especially if the pricing calculation is required to be done in real time.

2. In contrast to AC load flow, the sensitivities of line flows to nodal injections are constant in a DC model and dependent only on the network topology and parameters and not on the system operating point or loading. As will be seen later this attribute is particularly important as it greatly simplifies the computation of nodal prices.

4.2.2. Reference Network (mathematical model)

In order to calculate rapidly the critical loading of network branches, the algorithm uses three steps. First is to determine the intact system power flows at all branches in all periods. The power flow at branch A, which connects bus n to bus m, in the period ℓ can be calculated as follow. All power flows formulations in this section use per unit calculation.

$$F^0_{A,\ell} = x_A^{-1} . \sum_{i=1}^{NBus} \left[(X_{ni} - X_{mi}).P_{i,\ell} \right] \tag{4-1}$$

Where $F^0_{A,\ell}$ is the flow at branch A in the intact system of period ℓ

x_A is the reactance of branch A

X_{ni}, X_{mi} are the elements of the $[X]$ while $[X]$ can be expressed as $\left[\frac{\partial \boldsymbol{\theta}}{\partial \mathbf{P}}\right]$.

$P_{i,\ell}$ is the net injection (generation – demand) at bus i in period ℓ

NBus is the number of buses

Second is to calculate the flows at all branches in the credible contingent conditions for all periods. The power flow at branch A in the period ℓ when branch B is taken out from the system can be calculated using the following equation.

$$F^B_{A,\ell} = F^0_{A,\ell} + \lambda_{AB}.F^0_{B,\ell} \tag{4-2}$$

Where $F^B_{A,\ell}$ is the flow at branch A in the contingent system of period ℓ when branch B is out of service (contingency j). Branch B connects bus p to bus q.

$F^0_{B,\ell}$ is the flow at branch B in the intact system of period ℓ

λ_{AB} is the sensitivity of incremental flow at branch A in term of outage at branch B

The λ_{AB} can be calculated as follows.

$$\lambda_{AB} = \frac{x_B.(X_{np} - X_{nq} - X_{mp} + X_{mq})}{x_A.[x_B - (X_{pp} + X_{qq} - 2X_{pq})]} \forall A \neq B \tag{4-3}$$

$$\lambda_{AB} = -1 \ \forall A = B$$

Third is to find the critical loading for all branches across all considered network operation scenario. The critical loading of branch A can be obtained by finding the maximum flow at branch A across all considered network operation scenarios.

$$F_A^{rating} = \max(F^j_{A,\ell}) \ \forall \text{ j} \in \text{(0..NBranch) and } \ell \in \text{(1..NPeriod)} \tag{4-4}$$

Once the optimal rating of each plant is found, the comparison between the optimal reference network with the real network can be performed. By comparing those two networks, the optimality of various network components can be examined and measured accurately.

4.3. Allocation of optimal network costs

The methodology proposed and developed in this project is based on long run marginal cost pricing principles. In this implementation, application of marginal pricing principles leads to allocation of the cost of distribution network investments to users of the network on the basis contribution of nodal injections (effectively network users) to circuit critical flows. The developed framework captures accurately the spatial and temporal variation in demand rewarding those users whose network usage reduces the demand for new investment while penalising users driving investment. This pricing approach is particularly suitable for systems with distributed generation because the value of DG depends strongly on its location in the system and also varies in time.

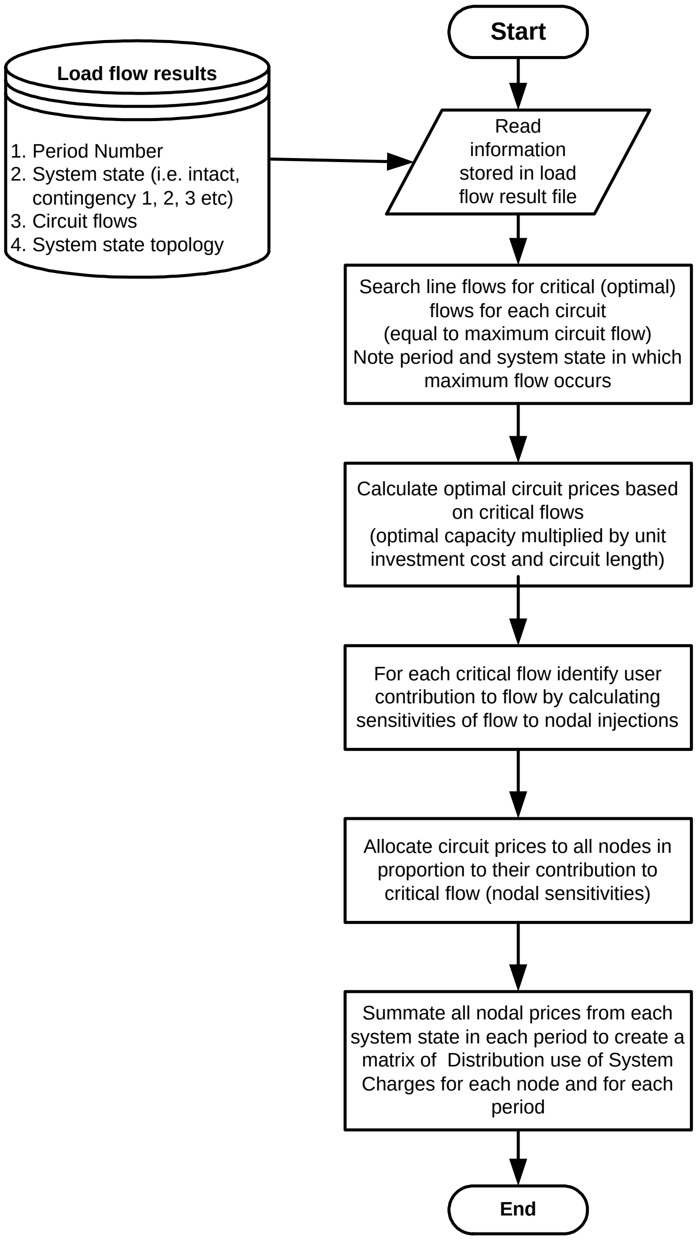

Figure 4-2 Steps in calculation of nodal prices

Figure 4-2 depicts the step-by-step process for calculating the optimal DUoS charges from the optimal network investments.

Temporal and spatial effects of load and generation are reflected in the charges, as the distribution use-of-system charges that are derived are specific to time-of-use as well as to location.

The process commences by reading the load flow results from which optimal capacities are determined. It is important to note that each critical circuit flow (maximum flow) is associated to a period as well as the system state in which it occurs. This association is crucial in determining time-of-use aspects of charges.

The optimal level of investment in the network is then calculated by multiplying the optimal circuit capacities with circuit lengths and unit investment costs or capacities with unit costs in the case of substations and transformers.

The next step in the calculation of nodal price is to identify the contribution of users (designated by node) to the critical flow. This is achieved by computing the sensitivities of circuit flow to nodal injections for the particular system state in which the critical flow occurs. This is a particularly novel and unique aspect of this approach as network security costs are allocated only to those users that benefit from security associated with the particular contingency in question. Most schemes in common use for allocating network security costs tend to be based on intact system network configuration. It can be shown that such schemes give rise to cross subsidies where users that do not motivate investment in network security are charged for these costs. Allocation of network security costs is dealt with simultaneously with intact system costs obviating the need to have a two-step process for cost allocation; one for intact system costs and another for costs of network security.

Once the sensitivities are known for each node, by dividing the capital and O&M cost of a circuit by its capacity, the annual or hourly circuit price can be derived in Pounds per unit of capacity i.e. £/kW/year or £/kWh. The nodal price is found by apportioning the circuit price in proportion to the sensitivities. This results in vectors of nodal prices for each period and system state for every node. The final nodal prices for each period are found by summing the nodal prices for each system state in each period.

4.3.1. Cost Allocation

Once the optimal ratings are known, the cost of that individual plant items can be calculated as follow (inside the square bracket is the unit of the variables).

$$\text{Cost}_A[\text{£/year}]=\text{Rating}_A[\text{MW}].\text{length}_A[\text{km}].\text{price}_A[\text{£/km/MW/year}] \qquad (4\text{-}5)$$

The cost of each branch then needs to be allocated to the appropriate loading periods when the critical loading occurs. If there is more than one critical period the cost of that plant item is spread uniformly to all correspondence loading periods; i.e. CostA for period ℓ = CostA / the number of correspondence loading periods. Or as an alternative, the cost needs to be spread proportionally as a function of the duration of the correspondence loading periods. The function is given below.

$$\text{Cost}_{A,\ell}=\text{Cost}_A.\tau_\ell.(\sum_{\dagger\in ARP}\tau_\dagger)^{-1} \qquad (4\text{-}6)$$

Where τ_ℓ is the duration of period ℓ

ARP : All relevant periods.

Generally, the critical loading condition is found in the contingent conditions. If in one critical period there is more than one contingency determine the optimal rating of a plant, the allocated cost of that plant item in the relevance period needs to be distributed uniformly to all the associated network states. Since the contingent conditions are likely to be short and the duration of each contingency is likely difficult to be obtained in a precise manner, it seems that a uniform distribution is a fair method for allocating this cost. The formula for allocating the cost is given as follow.

$$\mathrm{Cost}^{B}_{A,\ell} = \mathrm{Cost}_{A,\ell} . NCont_{A,\ell}^{-1} \tag{4-7}$$

Where $\mathrm{Cost}^{B}_{A,\ell}$ is the cost allocated for branch A in period ℓ during an outage at branch B.

$NCont_{A,\ell}$ is the number of network states in period ℓ required to be considered for allocating the cost of branch A

For the states of the networks in period ℓ, which do not contribute to the optimal branch A capacity, $\mathrm{Cost}_{A,\ell}$ is equal to zero.

4.3.2. Time of Use and Location Specific (TULS) Charges

Once the cost allocation for all branches in period ℓ and system state B is obtained, the time of use location specific network tariff can be calculated. The tariffs (exit) are based on the contribution of the power loading from those nodes to the cost of the required reference network.

$$\mathrm{NC}_{i,\ell} = -\sum[(\alpha_{Ai} + \lambda_{AB}.\alpha_{Bi}).\mathrm{Cost}^{B}_{A,\ell}.(\tau_{\ell}.F_{AB,\ell})^{-1}] \;\; \forall \mathrm{A,B} \in \{\text{all branches}\} \tag{4-8}$$

Where $\mathrm{NC}_{i,\ell}$ is the TULS exit tariff for node i in period ℓ

α_{Ai} is the sensitivity of incremental power flow at branch A in the intact system in term of power injection at bus i. α_{Ai} can be formulated as follow.

$$\alpha_{Ai} = x_A^{-1}.(X_{ni} - X_{mi}) \tag{4-9}$$

λ_{AB} is the sensitivity of incremental flow at branch A in term of outage at branch B

$F_{AB,\ell}$ is the capacity required at branch A when branch B is out of service in period ℓ

TULS entry tariff has the same magnitude but different polarity with the TULS exit tariff for the same time of use and location.

It is important to note that the methodology guarantees that total revenue obtained from the TULS charges across all the periods always remunerates exactly the total cost of the reference network.

4.4. Sensitivity analysis for linking process

A method based on the sensitivity analysis has been developed to link the charges from the higher to the lower voltage network. The method is to calculate the sensitivity of power injection from each node to the power injection at balancing points in the intact system and to allocate the tariff attached to the balancing points to each node.

$$\mathrm{NC}_{i,\ell} = \mathrm{NC}^0_{i,\ell} + \sum \frac{\partial P_k}{\partial P_i} \mathrm{NC}_{k,\ell} \tag{4-10}$$

Where $\mathrm{NC}_{i,\ell}$ is the TULS exit tariff for node i in period ℓ taking into account the balancing point exit tariff

$\mathrm{NC}^0_{i,\ell}$ is the TULS exit tariff for node i in period ℓ before taking into account the balancing point exit tariff

$\frac{\partial P_k}{\partial P_i}$ is the sensitivity of power injection at the balancing point k in the intact system in term of power injection at bus i.

$\mathrm{NC}_{k,\ell}$ is the balancing point exit tariff

4.5. Revenue reconciliation

For various reasons an economically optimal distribution network is practically impossible to be achieved therefore revenue reconciliation is an important and inevitable aspect of network pricing. Some of the main reasons that render achievement of optimal networks difficult in practice are: lumpiness of investment, economies of scale, standard line and cable conductor sizes (the optimal network is calculated assuming the capacity of circuit is a continuous variable), load and generation forecast uncertainty etc.

Apart from the difficulty of achieving optimal capacities in practice, there are certain cost elements associated with the operation and management of distribution systems that are independent of network capacity. These costs can only be recovered through revenue reconciliation. Examples of capacity independent costs include overheads and taxes.

Revenue reconciliation aims to balance revenue requirements against economic efficiency. In other words approved revenue targets should be achieved with as little deviation as possible on economic signals.

Some general methodologies for solving the revenue reconciliation problem, such as Ramsey pricing and the method of least squares are described in the classical book “Spot Pricing of Electricity” [5]. In recent years some researchers have devoted some effort to the development of revenue reconciliation methods specifically for optimal transmission pricing. For example Perera [6] propose a method in which the optimal prices are adjusted within indifference intervals over which network users are insensitive to transmission price. Wijayatunga et al. [7] have developed another revenue reconciliation method in which the transmission annuitised line investment cost K_l (£/MW.km.year) is modified until the value used in the calculation fully recovers the total investment. One of the problems with these methods is the tendency to penalise those users who are least sensitive to price.

Three such methods are presented and discussed in this report: (i) a method that adjusts the per unit cost of individual; plant such that the revenue obtained for each individual plant item matches the revenue required (ii) a method that uses multiplicative factor used to scale all charges such that the overall revenue received equals revenue required; (iii) a method that uses an additive factor such that charges are all shifted in such a way that the overall revenue is recovered and the price differentials between various locations are maintained.

Revenue reconciliation could be done for each voltage level independently so as to prevent cross-subsidisation between different voltage levels.

4.5.1. Method 1: adjustment of individual unit cost

The following revenue reconciliation method multiplies the individual annuitised reinforcement costs such that the total revenue from the charges to all customers associated to the assets is equal to the cost of the existing assets.

$$\mathrm{NC}_{i,\ell} = -\sum[(\alpha_{Ai} + \lambda_{AB}.\alpha_{Bi}).\kappa_{\mathrm{A}}.\mathrm{Cost}_{A,\ell}^{B}.(\tau_{\ell}.F_{AB,\ell})^{-1}]\ \forall \mathrm{A,B} \in \{\text{all branches}\} \tag{4-11}$$

Where $\mathrm{NC}_{i,\ell}$ is the TULS exit tariff for node i in period ℓ

α_{Ai} is the sensitivity of incremental power flow at branch A in the intact system in term of power injection at bus i. α_{Ai} can be formulated as follow.

$$\alpha_{Ai} = x_A^{-1}.(X_{ni} - X_{mi}) \tag{4-12}$$

λ_{AB} is the sensitivity of incremental flow at branch A in term of outage at branch B

κ_{A} is the ratio between the cost of existing plant and the cost of reference network

$F_{AB,\ell}$ is the capacity required at branch A when branch B is out of service in period ℓ

Despite being cost reflective and free cross subsidy, this method is not based on the reference optimal network and hence the allocation of network costs is biased to the existing network capacity and no longer reflecting the need of capacity driven by network customers.

4.5.2. Method 2: adjustment of TULS charges using a multiplicative factor

The second method uses a time specific scaling factor for both generation and demand charges to modify the amount of revenue earned from DUoS exit and entry charges such that the total revenue is equal to the proportion of the cost of existing assets which needs to be recovered. The formulation is given as follow:

$$\mathrm{NC}_{i,\ell}^{\mathrm{new}} = \mathrm{NC}_{i,\ell}^{old} \cdot (C_{\mathrm{R},\ell} \cdot C_{O,\ell}^{-1}) \tag{4-13}$$

Where $\mathrm{NC}_{i,\ell}^{\mathrm{new}}$, $\mathrm{NC}_{i,\ell}^{old}$ are the TULS exit tariff for node i in period ℓ after and before revenue reconciliation respectively

$C_{\mathrm{R},\ell}$ is the target revenue in period ℓ

$C_{O,\ell}$ is the cost of reference system recovered in period ℓ

4.5.3. Method 3: adjustment of TULS charges using an additive coefficient

Instead of using a scaling factor, the third method uses a time specific shifting factor for generation and demand charges to modify the amount of revenue earned from DUoS exit and entry charges. The formulation is given as follow:

$$\mathrm{NC}_{i,\ell}^{\mathrm{new}} = \mathrm{NC}_{i,\ell}^{old} + (C_{\mathrm{R},\ell} - C_{O,\ell}) \cdot (\sum_{i=1}^{NBus} (Pd_{i,\ell} + Pg_{i,\ell}) \cdot \tau_\ell)^{-1} \tag{4-14}$$

Where $\mathrm{NC}_{i,\ell}^{\mathrm{new}}$, $\mathrm{NC}_{i,\ell}^{old}$ are the TULS exit tariff for node i in period ℓ after and before revenue reconciliation respectively

$C_{\mathrm{R},\ell}$ is the target revenue in period ℓ

$C_{O,\ell}$ is the cost of reference system recovered in period ℓ

This method preserves the network tariff differences between participants in the energy market. This aspect could be crucial as it may affect competitiveness of network users in the electricity market.

The implementation of method 2 and 3 can be varied by applying different target revenue for each group of customers if necessary. For example: a different factor for demand and generation customers can be used for adjusting the tariffs to achieve a pre-defined target revenue. In addition, target revenue for different classes of demand and generation customers can also specified resulting different revenue reconciliation factors for different categories of customers.

5. Evaluation of charging methodology in a microgrid

This chapter describes four case studies that were conducted to illustrate the implementation of the proposed pricing approach into a microgrid connected to a public electricity system. The first case study demonstrates the computation of time of use and location specific DUoS tariffs. The second study investigated the impact of different installed capacity of micro sources connected to the microgrid on the DUoS tariffs and charges. The third case study illustrates the revenue reconciliation process to recover the allowable revenue. In the end of this chapter, the linking process of DUoS Charges for the public electricity system to the DUoS charges in the microgrid is illustrated.

5.1. Data for LV test system

All of the studies were performed on the developed "Study-Case LV Network" [7] as a test system. The Low Voltage (LV) network single line diagram is presented in Figure 5-1.

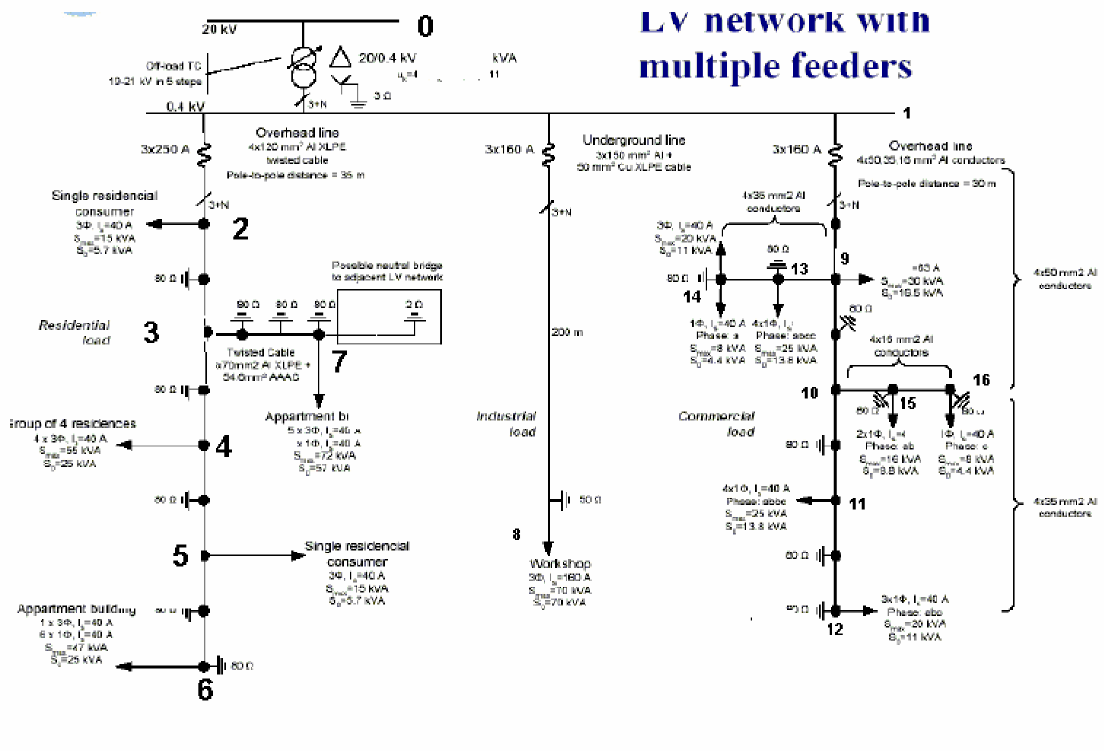

Figure 5-1 Single line diagram of the LV network study case.

This LV test system consists of three feeders. The leftmost feeder is a residential feeder, the middle feeder is an industrial feeder and the rightmost feeder is a commercial feeder. Each feeder then has different load characteristics.

[7] System characteristics are detailed in the document Study-Case LV Network.pdf by Stavros Papathanassiou

Demand data for maximum and minimum demand conditions for each node in the microgrid are presented in Table 5-1. In the minimum demand condition, demand is assumed to be 25% of the peak demand. Total maximum demand in peak and off peak conditions are 223 kW and 55.76 kW respectively.

Table 5-1 Maximum and minimum demand scenarios

Bus	Demand (kW)		Bus	Demand (kW)	
	Maximum	Minimum		Maximum	Minimum
1	0.00	0.00	9	14.03	3.51
2	4.80	1.20	10	0.00	0.00
3	0.00	0.00	11	11.73	2.93
4	19.36	4.84	12	9.35	2.34
5	4.80	1.20	13	11.73	2.93
6	11.99	3.00	14	13.09	3.27
7	47.97	11.99	15	7.48	1.87
8	63.00	15.75	16	3.74	0.94

Table 5-2 Network data

Node i	Node j	R	X	Length (km)	Price (£/kW/km/year)	Installed capacity (kW)	Annuitised Cost (£/year)
BUS 1	BUS 2	0.000010	0.000010	0.035	100.00*	170.0	595
BUS 1	BUS 8	0.033125	0.008750	0.200	100.00	110.0	2,200
BUS 1	BUS 9	0.007500	0.005000	0.030	100.00	110.0	330
BUS 17**	BUS 1	0.001150	0.003830	0.000	12.00	400.0	4,800
BUS 2	BUS 3	0.012500	0.003750	0.035	100.00	170.0	595
BUS 3	BUS 4	0.012500	0.003750	0.035	100.00	170.0	595
BUS 3	BUS 7	0.021870	0.004380	0.035	100.00	60.0	210
BUS 4	BUS 5	0.012500	0.003750	0.035	100.00	170.0	595
BUS 5	BUS 6	0.012500	0.003750	0.035	100.00	170.0	595
BUS 9	BUS 10	0.015000	0.010630	0.030	100.00	110.0	330
BUS 9	BUS 13	0.010630	0.005630	0.030	100.00	70.0	210
BUS 10	BUS 11	0.021250	0.005630	0.030	100.00	70.0	210
BUS 10	BUS 15	0.023130	0.006250	0.030	100.00	50.0	150
BUS 11	BUS 12	0.021250	0.005630	0.030	100.00	70.0	210
BUS 13	BUS 14	0.010630	0.005630	0.030	100.00	70.0	210
BUS 15	BUS 16	0.023130	0.006250	0.030	100.00	50.0	150
						Total cost	11,985

* Estimate annuitised cost of £ 10/kW for an average 100 m of LV circuit
** Bus 17 is a balancing point and connects to the public distribution system

Network data is presented in Table 5-2. Network is radial therefore R(resistance) and X(reactance) parameters are not significant to determine the critical flows. In this example, there is no contingency list since any contingency will split the microgrid into several islands. For simplicity, this example does not take into account the possibility that the microgrid can be split into more isolated independent systems.

In Table 5-2, the length and existing capacity of each line and the annuitised marginal cost to reinforce the lines are given. By multiplying the length with the annuitised price and the existing installed capacity, the annuitised cost for each line is calculated. This information is important and can be used as a basis for calculating target revenue in the revenue reconciliation process, which will be discussed later on.

It is important to note, that the figures given in this exercise are only for illustrative purposes.

Location of micro sources in the test system is shown in Figure 5-2.

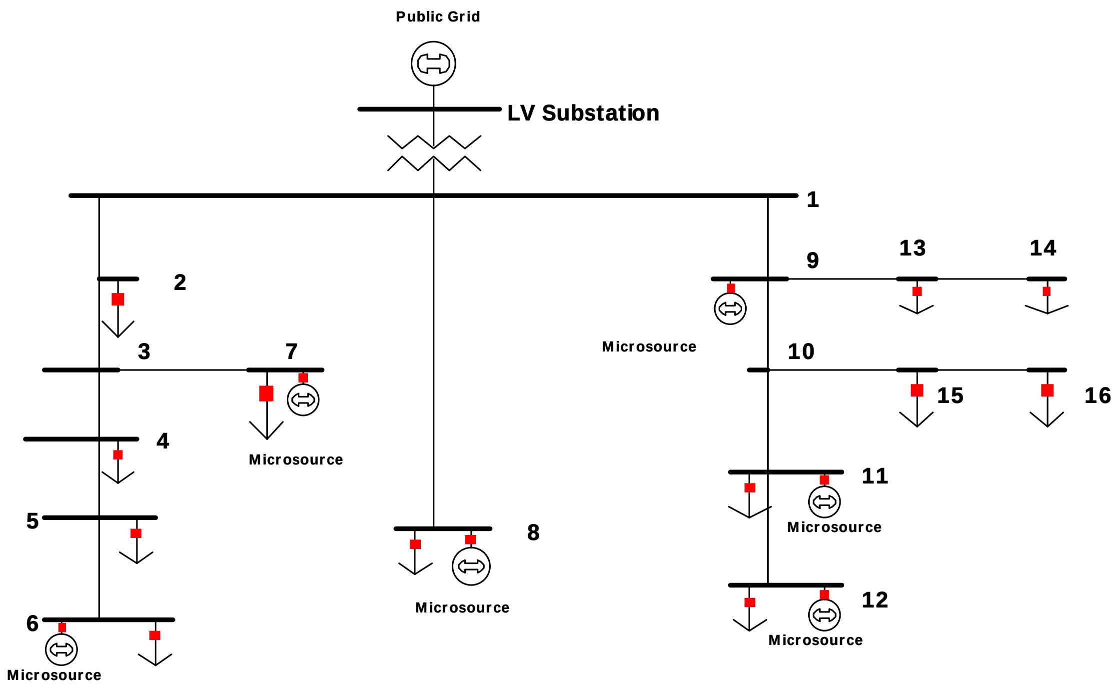

Figure 5-2 The LV microgrid test system

5.2. Case study I: computation of time of use and location specific DUoS charges for microgrid customers

Generation data used for this case study is presented in Table 5-3.

Table 5-3 Generation data for scenario I

Generator buses	Effective contribution during maximum peak loading condition	Installed capacity (kW)
BUS 6	5.5	11.0
BUS 7	5.5	11.0
BUS 8	9.5	19.0
BUS 9	2.0	4.0
BUS 11	3.0	6.0
BUS 12	3.0	6.0

The critical network loading is found by comparing the flow magnitude in the two loading conditions. If the critical flow is found in the peak loading condition, the type of asset will be classified as demand dominated (DD) whereas if the critical flow is driven by generation then the asset will be classified as generation dominated (GD) asset. If the flows in two loading condition are nearly the same then it can be classified as balance (BB) assets. Flows in the two loading scenarios, the critical flows and types of assets obtained from the computation are shown in Table 5-4.

Table 5-4 Optimal branch network capacity and classification of network assets

Branch	Node i	Node j	MW Flow$_{ij}$ (Peak Demand)	MW Flow$_{ij}$ (Peak Generation)	Critical flow (MW)	Installed Capacity (MW)	Type of assets	Cost (£/year)
1	BUS 1	BUS 2	77.91	0.23	77.91	170.0	DD	272.67
2	BUS 1	BUS 8	53.50	-3.25	53.50	110.0	DD	1,070.00
3	BUS 1	BUS 9	63.14	1.79	63.14	110.0	DD	189.44
4	BUS 17	BUS 1	194.55	-1.24	194.55	400.0	DD	2,334.61
5	BUS 2	BUS 3	73.11	-0.97	73.11	170.0	DD	255.88
6	BUS 3	BUS 4	30.64	-1.96	30.64	170.0	DD	107.25
7	BUS 3	BUS 7	42.47	0.99	42.47	60.0	DD	148.63
8	BUS 4	BUS 5	11.29	-6.80	11.29	170.0	DD	39.51
9	BUS 5	BUS 6	6.49	-8.00	8.00	170.0	GD	28.01
10	BUS 9	BUS 10	26.30	-3.93	26.30	110.0	DD	78.90
11	BUS 9	BUS 13	24.82	6.21	24.82	70.0	DD	74.46
12	BUS 10	BUS 11	15.08	-6.73	15.08	70.0	DD	45.24
13	BUS 10	BUS 15	11.22	2.81	11.22	50.0	DD	33.66
14	BUS 11	BUS 12	6.35	-3.66	6.35	70.0	DD	19.05
15	BUS 13	BUS 14	13.09	3.27	13.09	70.0	DD	39.27
16	BUS 15	BUS 16	3.74	0.93	3.74	50.0	DD	11.22
							Total	4,747.8

Note: negative flow indicates that power flows from bus j to bus I

With relatively small installed capacity of micro sources, most of the assets are demand dominated. Only branch between bus 5 and 6 is generation dominated.

The information about optimal network capacity can be used to assess the adequacy of existing assets. In this case, the capacity of existing assets is large enough to cope with the two extreme loading scenarios.

Table 5-5 DUoS exit charges

Node i	Maximum Demand Minimum Generation Period			Maximum Generation Minimum Demand Period		
	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)
BUS 1	0.00	12.00	0.00	0.00	0.00	0.00
BUS 2	4.80	15.50	74.35	1.20	0.00	0.00
BUS 3	0.00	19.00	0.00	0.00	0.00	0.00
BUS 4	19.36	22.50	435.49	4.84	0.00	0.00
BUS 5	4.80	26.00	124.72	1.20	0.00	0.00
BUS 6	11.99	26.00	311.77	3.00	-3.50	-10.49
BUS 7	47.97	22.50	1,079.24	11.99	0.00	0.00
BUS 8	63.00	32.00	2,016.00	15.75	0.00	0.00
BUS 9	14.03	15.00	210.38	3.51	0.00	0.00
BUS 10	0.00	18.00	0.00	0.00	0.00	0.00
BUS 11	11.73	21.00	246.33	2.93	0.00	0.00
BUS 12	9.35	24.00	224.40	2.34	0.00	0.00
BUS 13	11.73	18.00	211.14	2.93	0.00	0.00
BUS 14	13.09	21.00	274.89	3.27	0.00	0.00
BUS 15	7.48	21.00	157.08	1.87	0.00	0.00
BUS 16	3.74	24.00	89.76	0.94	0.00	0.00
BUS 17	0.00	0.00	0.00	0.00	0.00	0.00
		Total	5,455.55		Total	-10.49

Table 5-6 DUoS entry charges

Node i	Maximum Demand Minimum Generation Period			Maximum Generation Minimum Demand Period		
	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)
BUS 1	0.00	-12.00	0.00	0.00	0.00	0.00
BUS 2	0.00	-15.50	0.00	0.00	0.00	0.00
BUS 3	0.00	-19.00	0.00	0.00	0.00	0.00
BUS 4	0.00	-22.50	0.00	0.00	0.00	0.00
BUS 5	0.00	-26.00	0.00	0.00	0.00	0.00
BUS 6	5.50	-26.00	-143.00	11.00	3.50	38.50
BUS 7	5.50	-22.50	-123.75	11.00	0.00	0.00
BUS 8	9.50	-32.00	-304.00	19.00	0.00	0.00
BUS 9	2.00	-15.00	-30.00	4.00	0.00	0.00
BUS 10	0.00	-18.00	0.00	0.00	0.00	0.00
BUS 11	3.00	-21.00	-63.00	6.00	0.00	0.00
BUS 12	3.00	-24.00	-72.00	6.00	0.00	0.00
BUS 13	0.00	-18.00	0.00	0.00	0.00	0.00
BUS 14	0.00	-21.00	0.00	0.00	0.00	0.00
BUS 15	0.00	-21.00	0.00	0.00	0.00	0.00
BUS 16	0.00	-24.00	0.00	0.00	0.00	0.00
BUS 17	0.00	0.00	0.00	0.00	0.00	0.00
		Total	-735.75		Total	38.5

Table 5-7 Total charges for demand and generation customers

Group of customers	Total Charges in Maximum Demand Minimum Generation Period (£/year)	Total Charges in Maximum Generation Minimum Demand Period (£/year)	Subtotal (£/year)
Demand	5,455.55	-10.49	5,445.06
Generation	-735.75	38.5	-697.25
Total	4,719.8	28.01	4,747.81

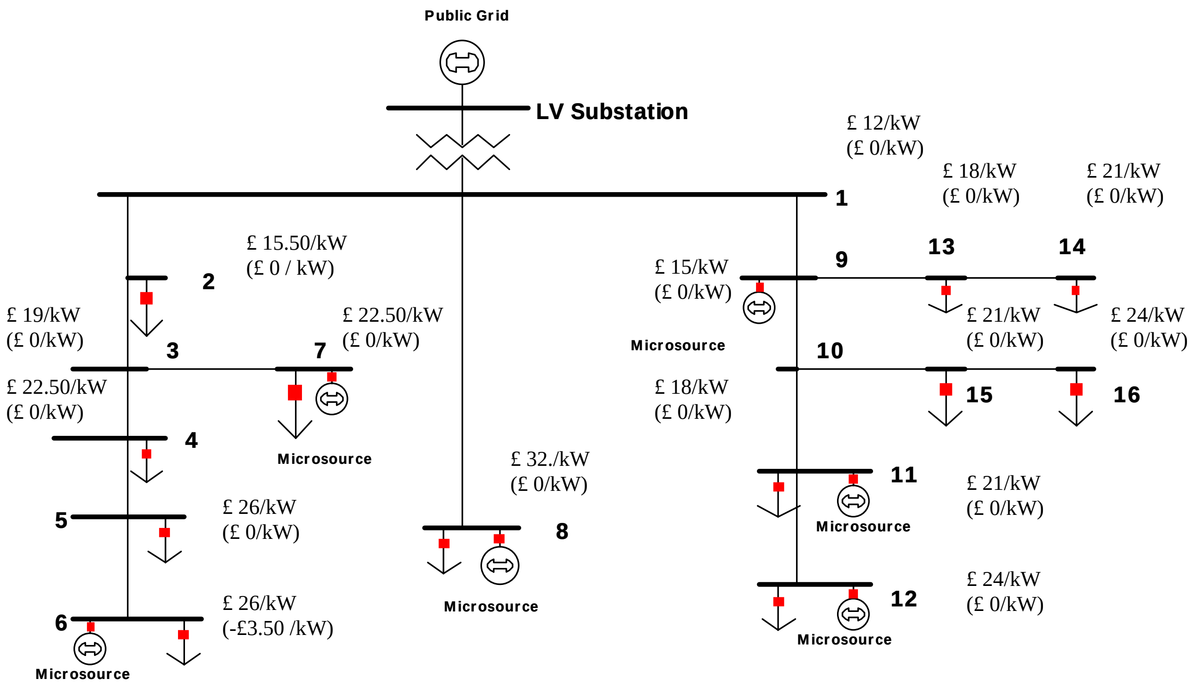

Figure 5-3 Time of use and location specific DUoS Exit tariffs placed at every nodes

Time of use and location specific (TULS) DUoS exit and entry charges for demand and generation customers across the microgrid are presented respectively in Table 5-5 and Table 5-6. Total charges for demand and generation customers in each period are summarised in Table 5-7 and Figure 5-3 shows pictorially TULS DUoS exit tariffs for each node in the microgrid. The first figure (without bracket) is the tariff in the period of maximum demand and minimum generation whereas the second figure (inside the bracket) is the tariff in the period of minimum demand and maximum generation. A DUoS entry tariff has the same magnitude but different polarity with a DUoS exit tariff at the same node.

From this case study, we can observe that:

1. Demand customers located further from the LV substation pay higher tariffs. These customers require more demand-dominated assets to supply their load compared with the customers located close to the LV substation.

2. Generation customers are rewarded by being paid £ 735.75/year since they reduce the demand of distribution network capacity during the maximum demand and minimum generation period. At the same period, demand customers pay £ 5,455.55/year.

3. In the period of maximum generation and minimum demand, most of nodal tariffs are zero since most of the network assets are demand dominated and hence recovered in the peak demand period. The generation customer at bus 6 pays £ 38.50/year to recover the cost of branch 9, which connects bus 6 to bus 5. While the demand customer at bus 6 is rewarded £ 10.49/year since he/she reduces the demand capacity of the branch 9.

4. The revenue obtained in the period of maximum demand and minimum generation is exactly recovering the cost of demand dominated assets for optimal network capacity whereas the revenue obtained in another period recovers the cost of generation dominated assets. The total revenue recovers precisely the cost of optimal capacity network. It is important to note that the cost of optimal capacity network is much lower than the cost of existing assets.

5. The total of demand charges is higher than the cost of the reference network since demand customers actually pay the reward for generation customers. This can be acceptable since without the generation customers, demand customers still need to pay the same amount of charges due to the larger network capacity required.

5.3. Case study II: Impact of different capacity of micro sources connected to microgrids on the DUoS tariffs and charges.

In order to examine the impact of various levels of micro sources penetration in the microgrid test system, three scenarios were developed. In scenario I, the total generation is 57 kW (around 25% of total peak demand). In scenario II and III, the total generation is increased to about 50% (112 kW) and 75% (167 kW) of total peak demand respectively. It is assumed that micro sources are micro CHPs and have 50% effective capacity contribution.

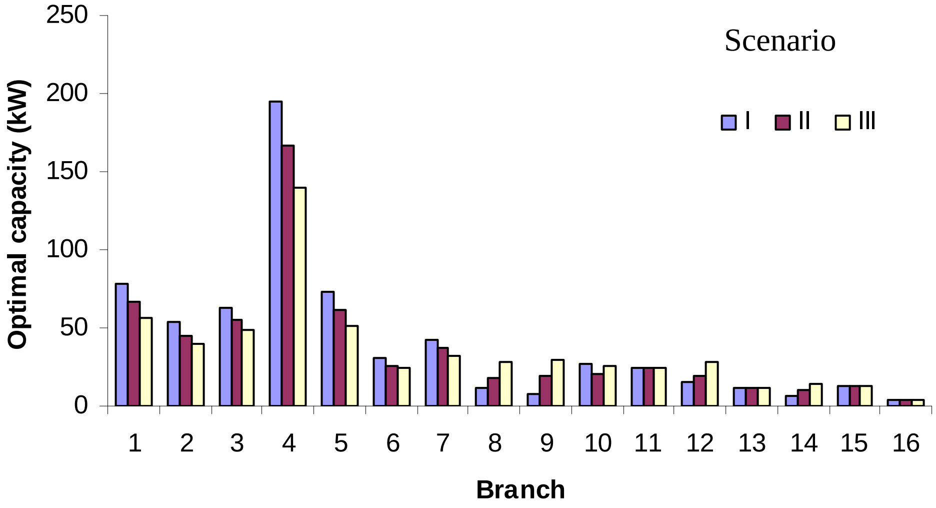

Figure 5-4 Impact of different penetration of micro sources on the optimal network capacity

Figure 5-4 shows that the capacity of micro sources can actually displace the capacity of some network assets as indicated by the reduction of the required optimal capacities for branches 1 to 7 in scenario II and III. However, installation of larger generation capacity does not necessarily always reduces the demand of network capacity as demonstrated by the increase of demand for capacity for branches 8,9,12, and 14 in scenario II and III. Fluctuation of optimal capacity at branch 10 is particularly interesting since it decreases in scenario II but then increases in scenario III when the capacity of the asset is driven no longer by demand but by generation.

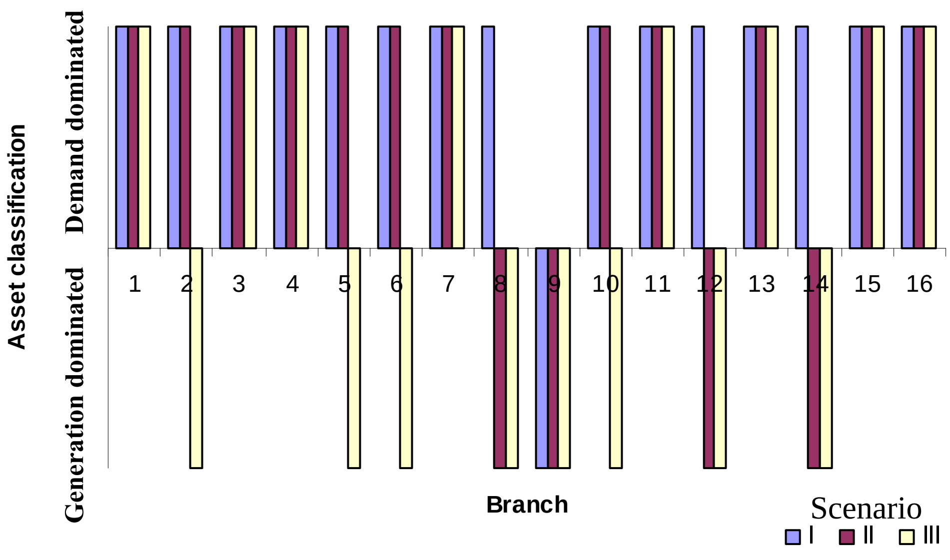

Figure 5-5 Impact of different penetration of micro sources on the asset classification

The increase of micro sources' installed capacity also changes the classification of various assets. With most of the assets are demand dominated in scenario I, some assets become generation dominated in scenario II and III as illustrated in Figure 5-5.

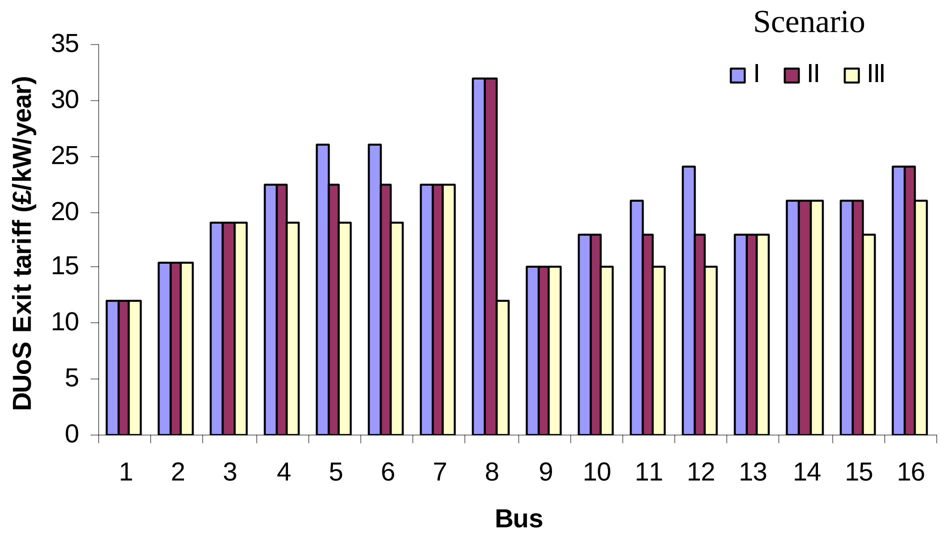

Figure 5-6 Impact of different penetration of micro sources on the DUoS exit tariffs in the peak demand period

The increase of installed micro sources capacity also reduces the DUoS exit tariffs at various locations in the microgrid as illustrated in Figure 5-6. This is because some assets become generation-dominated assets and are recovered through generation charges.

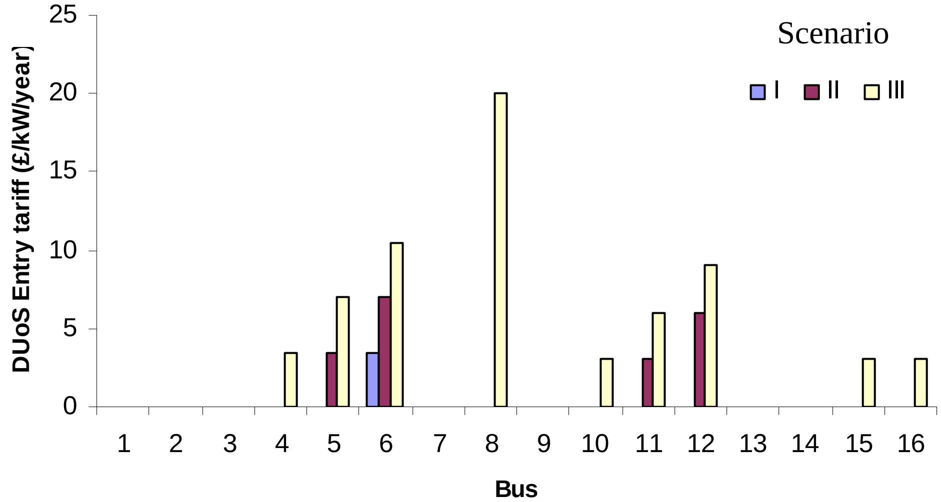

Figure 5-7 Impact of different penetration of micro sources on the DUoS entry tariffs in the off peak demand period

In contrast, Figure 5-7 demonstrates the increase of generation DUoS tariffs at various locations mostly at generator connected nodes when the installed capacity of micro sources becomes larger.

Hence, the charges for individual customers vary following the variation in the DUoS tariffs. The reduction of total charges for demand customers in scenario II and III is compensated by the increase of charges for generation customers in the respective periods. It is important to note that the changes are not linear to the changes in the installed micro sources capacity. Table 5-8 - Table 5-10 demonstrate the demand and generation charges for each scenario.

Table 5-8 DUoS charges in the period of maximum demand minimum generation

Group of customers	Scenario I	Scenario II	Scenario III
Demand	5,455.55	5,305.50	3,822.10
Generation	-735.75	-1347	-1,358.25
Total	4,719.80	3,958.50	2,463.85

Table 5-9 DUoS charges in the period of minimum demand maximum generation

Group of customers	Scenario I	Scenario II	Scenario III
Demand	-10.49	-48.01	-418.87
Generation	38.5	262	1,721.50
Total	28.01	213.99	1,302.63

Table 5-10 Sum of DUoS charges in both periods

Group of customers	Scenario I	Scenario II	Scenario III
Demand	5,445.06	5,257.49	3,403.23
Generation	-697.25	-1085	363.25
Total	4,747.81	4,172.49	3,766.48

Although the total amount of generation charges in the off peak demand period increases following the increase in the number of generation dominated assets, the total reward obtained by generation customers increases in scenario II. However, with the significant of increase in generation capacity, generation customers then need to pay positive charges as demonstrated in scenario III.

The total charges from all microgrid customers become smaller in scenario II and III indicating that the total cost of optimal network becomes less although some assets may need larger capacity as shown in Figure 5-4 previously.

5.4. Case study III: Revenue Reconciliation

This case study demonstrates the revenue reconciliation process to adjust DUoS charges such that the total charges are equal to the amount of money that needs to be recovered. The installed capacity of micro sources was set to be about 75% of total peak demand. This denotes the scenario III of the previous case study (II). The detail of the installed capacity and effective contribution of each micro source is presented in Table 5-11.

Table 5-11 Generation data for scenario III

Generator buses	Effective contribution during maximum peak loading condition	Installed capacity (kW)
BUS 6	16.5	33.0
BUS 7	16.5	33.0
BUS 8	28.0	56.0
BUS 9	5.5	11.0
BUS 11	8.5	17.0
BUS 12	8.5	17.0

Three revenue reconciliation methods described in section 4.5 were used for the study. Method 1 multiplies the individual annuitised reinforcement costs, method 2 uses a time specific scaling factor and method 3 uses a time specific shifting factor such that the total revenue from the charges to all customers associated to the assets is equal to the cost of the existing assets.

5.4.1. Demand charges

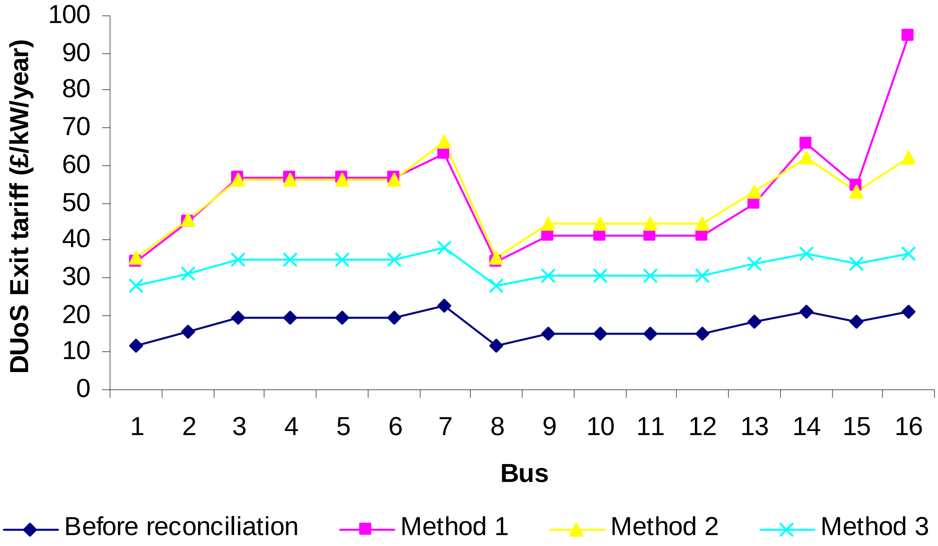

Figure 5-8 Impact of revenue reconciliation on the DUoS Exit tariff for peak demand period

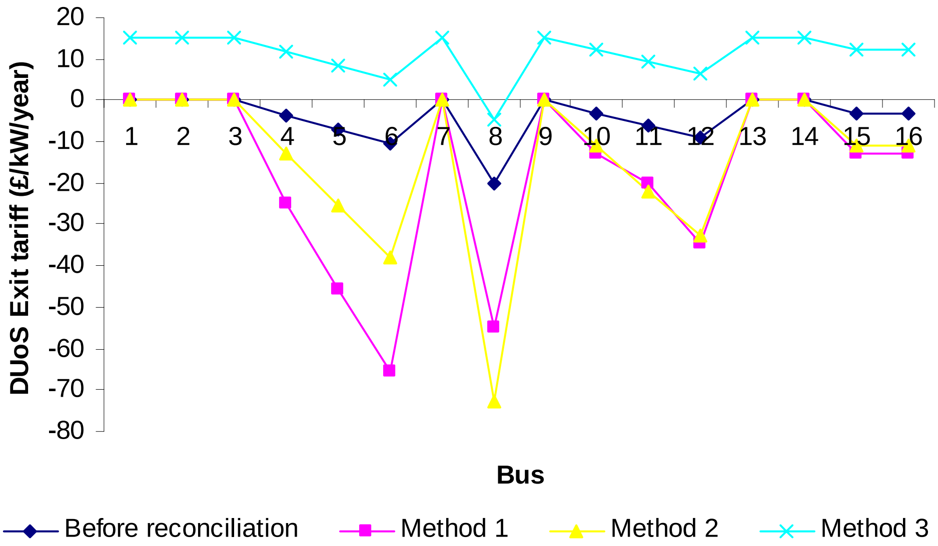

Figure 5-9 Impact of revenue reconciliation on the DUoS Exit tariff for off peak demand period

Figure 5-8 and Figure 5-9 illustrate the resultants of different revenue reconciliation methods. The profiles of tariffs are similar but the magnitude can be significantly different depending on the selected reconciliation method. It was noted that by using a scaling factor, the zero tariffs remain zero. However, method 3 shifts tariffs for demand customers using the same coefficient and hence the customers who initially get zero tariffs have non zero tariffs and vice versa. The polarity of tariffs can also be different when method 3 is used. Consequently, customers, who should get paid initially, may need to pay and vice versa. By using scaling methods, the tariff differences between customers are magnified while method 3 preserves the differences. This specific feature of method 3 may be desirable especially in the competitive environment of electricity market.

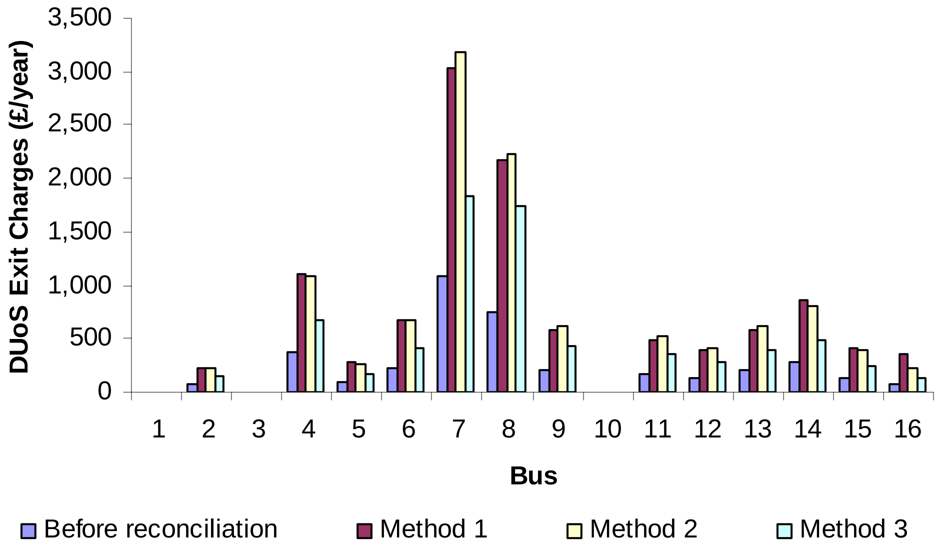

Figure 5-10 Impact of revenue reconciliation on the DUoS Exit charges for peak demand period

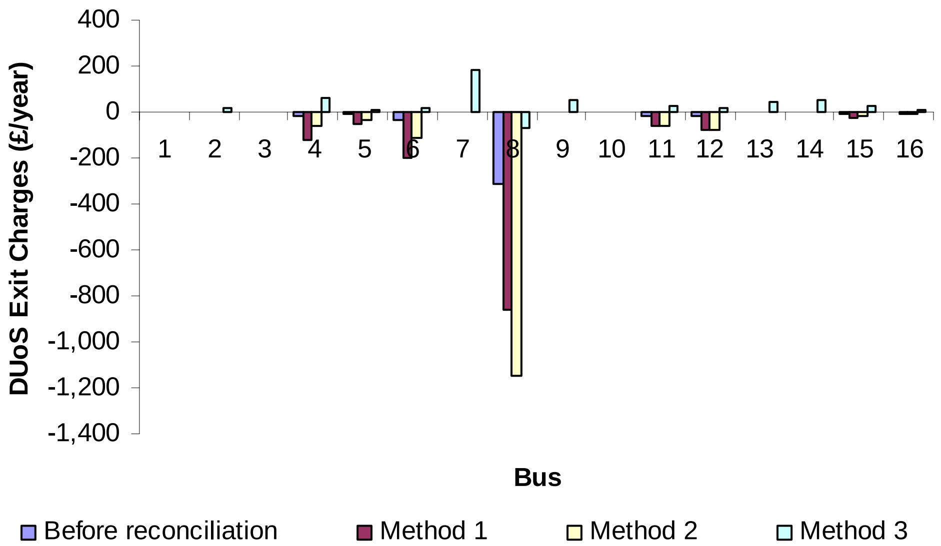

Figure 5-11 Impact of revenue reconciliation on the DUoS Exit charges for off peak demand period

Figure 5-10 - Figure 5-11 show the changes in demand charges after the revenue reconciliation. Depending on the method selected, the charges can be different. This emphasises the importance of selecting an appropriate revenue reconciliation method that can give the desired economic impacts.

5.4.2. Generation charges

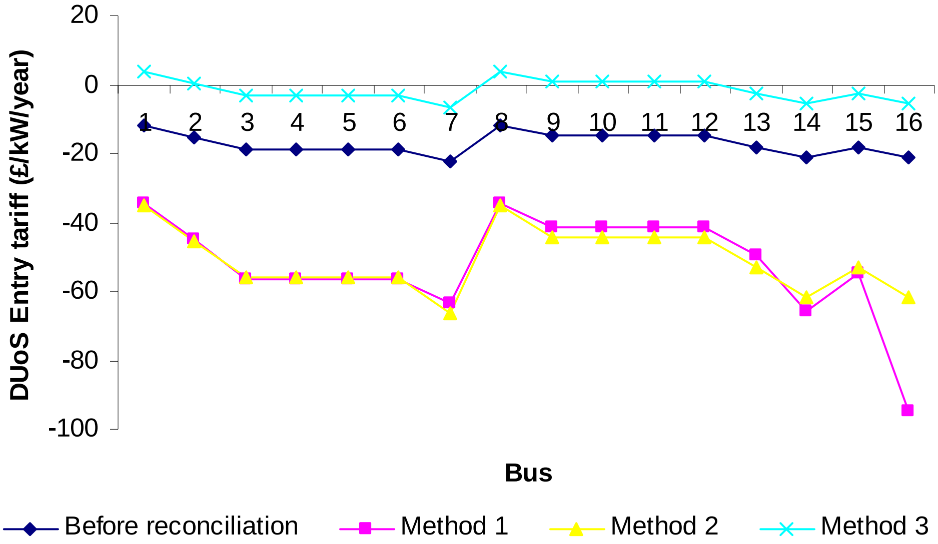

Figure 5-12 Impact of revenue reconciliation on the DUoS entry tariffs for peak demand period

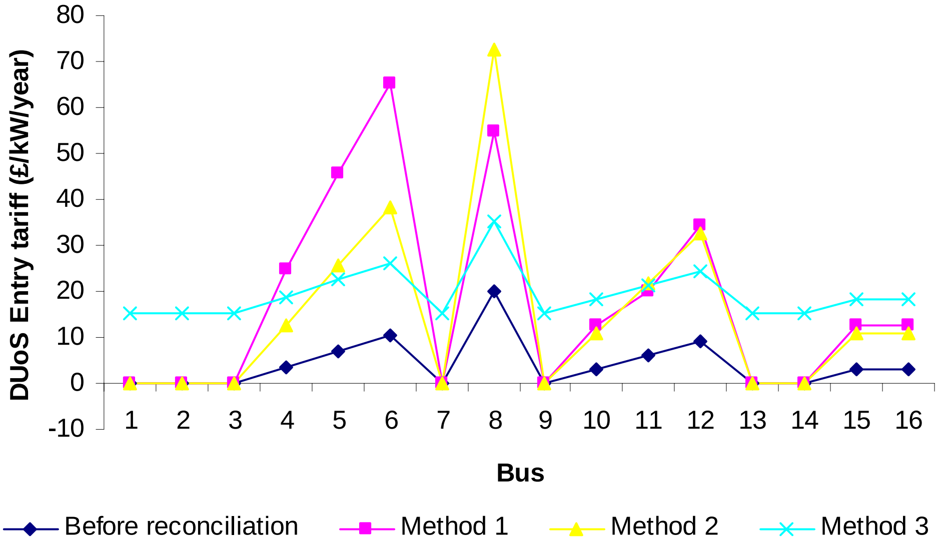

Figure 5-13 Impact of revenue reconciliation on the DUoS entry tariffs for off peak demand period

Figure 5-12 and Figure 5-13 illustrate the resultants of different revenue reconciliation methods on the generation DUoS tariffs. The use of scaling methods preserves the equality between the magnitude of demand and generation tariffs. In contrast, this equality cannot be possibly preserved by using the shifting method.

Figure 5-14 - Figure 5-15show the changes in generation charges after the revenue reconciliation.

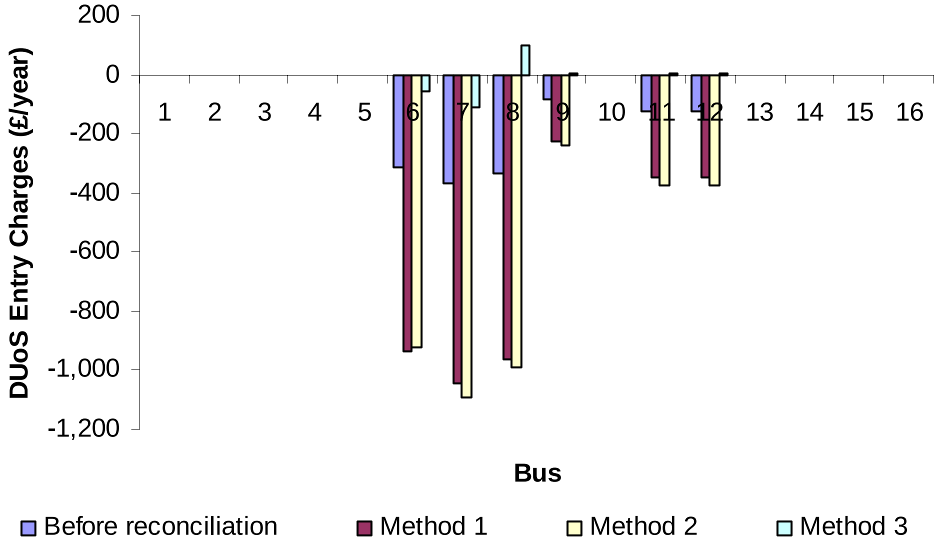

Figure 5-14 Impact of revenue reconciliation on the DUoS entry charges for peak demand period

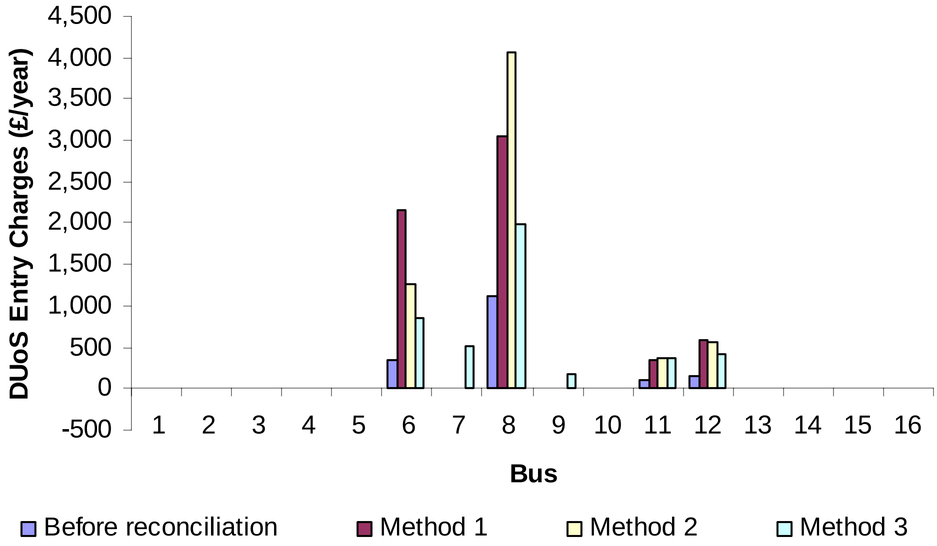

Figure 5-15 Impact of revenue reconciliation on the DUoS entry charges for off peak demand period

5.4.3. Recapitulation of demand and generation charges

Table 5-12 Impact of revenue reconciliation on the total of demand and generation charges for each period

Demand charges (£/year)	Method 1	Method 2	Method 3
Max demand min generation	11,120.22	11,246.73	7,304.57
Min demand max generation	-1,406.15	-1,522.50	440.34
Sub total	9,714.07	9,724.24	7,744.91
Generation charges (£/year)	Method 1	Method 2	Method 3
Max demand min generation	-3,870.22	-3,996.73	-54.57
Min demand max generation	6,141.15	6,257.50	4,294.66
Sub total	2,270.93	2,260.76	4,240.09
Total	**11,985.00**	**11,985.00**	**11,985.00**

Table 5-12 demonstrates the impact of different revenue reconciliation methods on the total of demand and generation for each period. It shows that the resultants of the first two methods are quite similar. It also shows that the cash flows in method 3 is relatively smaller than the flows in the first two methods. Irrespective of the revenue reconciliation method selected, the total charges are the same (£ 11,985/year) that recovers exactly the cost of existing network (Table 5-2).

5.5. Case study IV: Charges from upstream voltage networks

In addition to recover the network cost of the microgrid, microgrid customers also contribute on the cost of public electricity system to where the microgrid is connected during the grid connected mode. Hence, the DUoS charges should also include the charges for the relevant upper stream networks.

For this study, the tariff structure in Figure 5-16 was used. Figure 5-16 was obtained from the previous illustration explained in section 3.4. For simplicity, it was assumed that the connection of microgrid does not affect the obtained tariff structure.

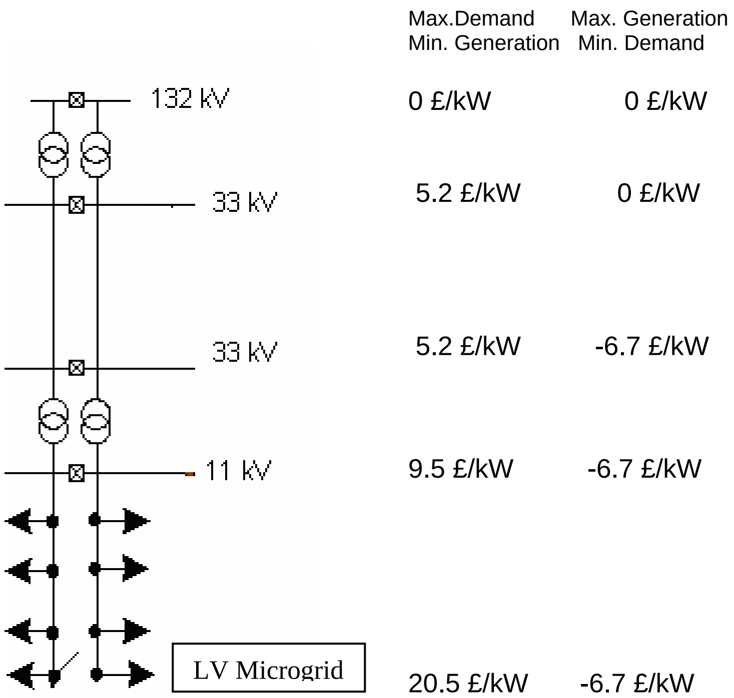

Figure 5-16 Evaluation of DUoS exit charges for the example given in Figure 3-4.

Therefore, the DUoS tariff for the balancing point of the microgrid was set to £ 20.5/kW in the peak demand period and - £6.7/kW in the off peak demand period. Since the microgrid is radial, the adjustment of tariffs can be straight forward. All peak demand and off peak demand DUoS exit tariffs were shifted by £ 20.5/kW and £-6.7/kW respectively. DUoS entry tariffs for peak and off peak demand periods were also shifted by -£20.5/kW and £6.7/kW respectively.

It is important to note that the upstream charges can be added after revenue reconciliation has been done. In this case study, the result of revenue reconciliation for method 2 was used. The final DUoS exit and entry charges for microgrids customers are demonstrated in Table 5-13 - Table 5-15.

Table 5-13 Final DUoS exit charges

Node i	Maximum Demand Minimum Generation Period			Maximum Generation Minimum Demand Period		
	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)
BUS 1	0.00	55.81	0.00	0.00	-6.70	0.00
BUS 2	4.80	66.11	317.13	1.20	-6.70	-8.03
BUS 3	0.00	76.41	0.00	0.00	-6.70	0.00
BUS 4	19.36	76.41	1,478.89	4.84	-19.42	-93.98
BUS 5	4.80	76.41	366.53	1.20	-32.14	-38.55
BUS 6	11.99	76.41	916.22	3.00	-44.87	-134.50
BUS 7	47.97	86.71	4,159.02	11.99	-6.70	-80.34
BUS 8	63.00	55.81	3,516.08	15.75	-79.40	-1,250.52
BUS 9	14.03	64.64	906.55	3.51	-6.70	-23.49
BUS 10	0.00	64.64	0.00	0.00	-17.60	0.00
BUS 11	11.73	64.64	758.21	2.93	-28.51	-83.60
BUS 12	9.35	64.64	604.37	2.34	-39.41	-92.13
BUS 13	11.73	73.47	861.76	2.93	-6.70	-19.65
BUS 14	13.09	82.29	1,077.23	3.27	-6.70	-21.93
BUS 15	7.48	73.47	549.53	1.87	-17.60	-32.92
BUS 16	3.74	82.29	307.78	0.94	-17.60	-16.46
BUS 17	0.00	20.50	0.00	0.00	-6.70	0.00
		Total	15,819.3		Total	-1,896.1

Table 5-14 Final DUoS entry charges

Node i	Maximum Demand Minimum Generation Period			Maximum Generation Minimum Demand Period		
	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)	Demand (kW)	Tariff (£/kW/year)	Charges (£/year)
BUS 1	0.00	-55.81	0.00	0.00	6.70	0.00
BUS 2	0.00	-66.11	0.00	0.00	6.70	0.00
BUS 3	0.00	-76.41	0.00	0.00	6.70	0.00
BUS 4	0.00	-76.41	0.00	0.00	19.42	0.00
BUS 5	0.00	-76.41	0.00	0.00	32.14	0.00
BUS 6	16.50	-76.41	-1,260.74	33.00	44.87	1,480.60
BUS 7	16.50	-86.71	-1,430.68	33.00	6.70	221.10
BUS 8	28.00	-55.81	-1,562.70	56.00	79.40	4,446.30
BUS 9	5.50	-64.64	-355.51	11.00	6.70	73.70
BUS 10	0.00	-64.64	0.00	0.00	17.60	0.00
BUS 11	8.50	-64.64	-549.43	17.00	28.51	484.66
BUS 12	8.50	-64.64	-549.43	17.00	39.41	670.04
BUS 13	0.00	-73.47	0.00	0.00	6.70	0.00
BUS 14	0.00	-82.29	0.00	0.00	6.70	0.00
BUS 15	0.00	-73.47	0.00	0.00	17.60	0.00
BUS 16	0.00	-82.29	0.00	0.00	17.60	0.00
BUS 17	0.00	-20.50	0.00	0.00	6.70	0.00
		Total	-5,708.49		Total	7,376.4

Table 5-15 Total charges for demand and generation customers

Group of customers	Total Charges in Maximum Demand Minimum Generation Period (£/year)	Total Charges in Maximum Generation Minimum Demand Period (£/year)	Subtotal (£/year)
Demand	15,819.30	-1,896.10	13,923.2
Generation	-5,708.49	7,376.40	1,667.91
Total	10,110.81	5,480.3	15,591.11

Table 5-15 recapitulates the total of demand and generation charges to the microgrid customers. With £ 11,985 charges are paid to recover fully the network cost of the microgrid, the rest of charges (£ 3606.11) is paid to recover partially the cost of public electricity system to where the microgrid is connected to.

6. Summary

This report has presented a novel framework for allocating security driven distribution network costs based on the notion of reference network and long run investment marginal cost pricing principles to derive cost reflective charging methodology. This methodology takes into account the spatial and temporal contribution for each network users on the demand of network capacity. The resultant of the methodology is time of use and location specific tariffs which overcomes the lack of cost reflectivity, the economic inefficiency and the inability of the present DUoS charging methodology to price the use of distribution system with distributed generation.

The methodology has been illustrated clearly and the required input data and mathematical formulation for efficient computation has been developed and explained in detail in this report. The formulation has been tested successfully in a number of conducted case studies using a LV microgrid system to illustrate and demonstrate the features of the proposed methodology.

Three revenue reconciliation methods and a method to include use of system charges from public electricity system to where microgrids are connected have also been developed, implemented and tested successfully. The evaluation of three revenue reconciliation methods was described in the case study. Each method may lead to different economic impacts to the network customers. And finally, the last case study demonstrated the ability of the proposed charging methodology to include the contribution of microgrid’s customers on the cost of the public electricity system. This is important since microgrids can be rewarded (get paid) for reducing the demand of network capacity in the public distribution and transmission system.

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Report

Allocation of Cost of Losses in Microgrids

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Table of Content

1. Introduction

Allocation of cost of losses has gained significant attention since the advent of competitive electricity markets. Although the amount of losses is relatively small, approximately 5%-8% of total energy consumption, the cost of losses allocated as charges to network customers can significantly influence the economic performance of their operation and investment decisions. Clearly, local micro sources have higher value in terms of loss reduction related benefits compared with large remote generators (central generators) Hence, the allocation of losses can have a direct impact on value of generation and this should be reflected in the corresponding revenues and profits.

Allocation of cost of losses is part of network pricing which should provide economic signals to network customers (both generation and demand) to stimulate economic efficiency in the system. Although it is simple, on one hand, to apply the same average charges to all customers, this will not stimulate economic efficiency since customers will not be receiving appropriate signals (penalties or rewards) for their impact on energy losses. Therefore, there would be no driver to influence customers' decisions to reduce network losses. On the other hand, the computation of charges taking into account temporal and spatial contribution of losses from each customer will provide commercial incentive to reduce losses in the system. The reduction can be achieved for example by installing new generators and demand in appropriate locations (investment decision), and adjust the patterns of consumption (generation) to reduce losses (operation decision). Although the signals will initially influence the individual customers, eventually the economic efficiency of the whole system will be improved.

In the scope of MICROGRIDS, the issues about allocating losses and the cost of losses need to be solved efficiently. Allocation of cost of losses as one of the components in network pricing to induce economic efficiency in microgrids is crucial. As microgrids represent micro scale power systems consisting of local and distributed micro sources connected directly to Low Voltage grids, controllable and fixed loads, storages and decentralised control architecture, the allocation of cost of losses in the microgrids becomes more challenging compared with the same problem for ordinary distribution networks without distributed generation.

Acknowledging the importance of allocating losses efficiently, it is unsurprising that many methodologies have been proposed to solve the allocation of losses and allocation of cost of losses problems. Some of the methodologies are the proportional methods, marginal losses coefficient methods, proportional-sharing methods, incremental losses allocation, allocation of losses based OPF and etc. A literature review about all of the cited methods was given in the MICROGRIDS reports by Saraiva, J.T et al [1], and Papadogiannis, K and Hatziargyriou, N. [2]. The methods above calculate and quantify the cost of losses that need to be paid by each customer indirectly by multiplying the losses allocated to the customers with the pre-defined unit cost of losses.

As contribution to MICROGRIDS project WPG task G4 [3], this report presents and describes an alternative approach for allocating losses and the related charges using an advance Optimal Power Flow technique, namely Primal Dual Interior Point

Method. The approach can be integrated with the energy market operation inside the microgrid and can be embedded directly into Micro Grid Central Controller (MGCC) to provide real time pricing signal. This approach also provides a solution to the deficiencies in other methods such as (i) results are sensitive to the choice of slack bus, (ii) inability to take into account the impact of reactive power flows in active losses, (iii) dynamic changes in generation patterns, (iv) network constraints and etc.

The structure of the report can be described as follows.

Chapter 2 describes the features of the proposed approach for allocating cost of losses directly into the nodal charges in the real time pricing of microgrid's short term energy market operation.

The problem described in this chapter is in essence an Optimal Power Flow problem. The problem is to minimise the total operation cost of short term energy market in microgrids taking into account generation bids, demand of supply, generation constraints, and network constraints including voltage and power flow constraints in the system. The optimal solution is achieved by despatching the merit order generation and by optimally controlling network devices to regulate voltage and flows to be between the permissible limits. The formulation of the problem is presented and described in section 2.3.

The OPF problem is then solved using an advance Non Linear Primal Dual Interior Point method (PDIPM). Section 2.4 describes the general formulation and the algorithm of the PDIPM.

Chapter 3 describes a number of case studies on a microgrid LV test system conducted to demonstrate the ability of the proposed approach to allocate the cost of losses efficiently and charges to the customers appropriately to recover not only the cost of energy taken by the load but also the cost of losses. Key observations of the main characteristics of the proposed approach including economic signals given to network users are presented and described. A novel losses indicator that can be derived easily by subtracting nodal charges with nodal charges at marginal generators is also proposed. This indicator provides a measure of network customers' contribution to losses.

Finally, a summary of the report is given in the end of the report.

2. Proposed approach

2.1. Advantages

In the absence of real time pricing, losses coefficients are generally computed ex ante the real time operation to provide economic signals to induce efficient system operation. Hence, generation despatches, load and network conditions need to be pre-defined (forecasted) to enable the computation of losses allocation. The resultant can then be used to adjust generation output to supply both loads and losses in power system. It can also be used to adjust the customer charges. Since losses are dynamically changing following the variations in generation, demand and network conditions, some adjustment then need to be done after the real time operation in order to provide exact losses allocation and the associated charges.

An alternative method is to calculate losses and allocate the cost of losses directly in the nodal charges concurrently in the real time operation. This task can be performed using an AC Optimal Power Flow. This method has several important advantages such as:

1. It omits the need for computing losses coefficients in the operational planning stages and the correction needed afterward.

2. It quantifies the monetary value of losses directly and provides real time economic signals to promote short-term efficient operation. This quantification is typically not provided by non-OPF methods. In order to determine the losses related charges for non-OPF methods, the unit cost of losses is generally approximated. This approximation is prone to error due to the non-linearity of incremental cost in generation.

3. It omits the need for the reference bus and its impact on losses allocation. Non-OPF methods use a pre-defined reference bus as a slack bus to supply the imbalance of power in the system. As generation dispatch must be given ex ante and fixed during the computation, the imbalance of power due to losses is supplied from a slack bus, which may not be appropriate (correct). Therefore, the choice of slack bus is crucial for non-OPF methods. Some mechanisms have been developed to neutralise the impact of selecting different slack bus in the losses allocation. In contrast, OPF methods compute the optimal dispatch and determine automatically the marginal generators taking into account network and generation constraints. OPF methods can also handle directly conditions with more than one marginal generator in the system. These conditions occur especially when the network is congested.

4. It can recognise the impact of reactive power flows on active power losses. Most of the losses allocation methods especially based on DC calculation ignore this impact.

2.2. Implementation issues

AC OPF problem is complex non linear problem. The consistencies, accuracies and robustness of AC OPF solution methods have also been significantly improved during the last decade. This area of optimisation in power system has also received great attention and development. One of the advance methods that will be described later on in this report is the non-linear interior point method.

OPF has been used widely for optimising power system operation in transmission level. However, its application has not received the same degree of attention in distribution systems mainly because the distribution systems are still operated passively ("fit and forget approach"). With the foreseeable penetration of distributed generation and microgrids in future, the operation of distribution networks will need to be more actively control to get the maximum investment and operation performance of the network. This operation will mimic the operation in transmission level.

Microgrids can be seen as small scale power systems and therefore need to be actively controlled to enable secure and optimal operation. Active control is particularly needed when microgrids are operated in islanding mode. Microgrids can also form commercial boundaries between public electricity system and microgrids, hence there is a need for active control of market operation in addition to system technical operation. In the framework of microgrids control strategy in MICROGRIDS project, this control requirement will be handled by using MicroGrid Central Controller (MGCC). Therefore, the implementation of the proposed allocation of losses and the related charges can be directly included in the MGCC especially if the MGCC uses OPF engine to compute real time pricing information and uses the information to induce economically efficient long and short term market and system operation.

2.3. Formulation of OPF problem

The objective function of the OPF problem is to minimise the total operation cost of microgrids including the cost of active and reactive power generation subject to the power balance, the physical and operation constraints for each device in the system including microgrid constraints (voltage and thermal constraints). It is important to note that the OPF problem does not minimises losses explicitly but minimises the cost of purchasing energy that includes losses. Consequently, the solution can be different from the solution obtained by minimising losses directly. In the former case, the amount of losses can be higher compared with the later since merit generators can be located remotely from the load; however, the operation costs will be smaller.

In order to simplify the problem, a fixed load and a generator can be used to form a controllable load in microgrids. Both of fixed load and generator have the same capacity as the maximum load of the controllable load. Therefore, the load can be controlled to vary from zero load to the maximum load. Non-despatchable generators and non-controllable loads are considered as fixed negative and positive loads respectively. The constraints of these units are excluded from the OPF problem.

The problem can be formulated as follows.

Objective function

$$\text{Minimise } \Psi = \sum_{i=1}^{\text{MN}} (C_{p,i}^{2} P_{g,i} + C_{p,i}^{1} P_{g,i} + C_{p,i}^{0}) + \sum_{i=1}^{\text{MN}} (C_{q,i}^{2} Q_{g,i} + C_{q,i}^{1} Q_{g,i} + C_{q,i}^{0}) \quad \textbf{(2.1)}$$

Subject to[1]

1. Active power balance equation

$$P_g - P_d - \sum_{\substack{j=0 \\ j\neq i}}^{MN} \mathrm{FP_{ij}} = 0 \qquad \textbf{(2.2)}$$

2. Reactive power balance equation

$$Q_g - Q_d - \sum_{\substack{j=0 \\ j\neq i}}^{MN} \mathrm{FQ_{ij}} = 0 \qquad \textbf{(2.3)}$$

3. Voltage limit

$$\mathrm{V_{min}} \leq V \leq \mathrm{V_{max}} \qquad \textbf{(2.4)}$$

4. Limits of active and reactive power generation

$$\mathrm{P_{gmin}} \leq P_g \leq \mathrm{P_{gmax}} \qquad \textbf{(2.5)}$$

$$\mathrm{Q_{gmin}} \leq Q_g \leq \mathrm{Q_{gmax}} \qquad \textbf{(2.6)}$$

5. Limits of transmission control devices (tap changers, shunts)

$$\mathrm{t_{min}} \leq t \leq \mathrm{t_{max}} \qquad \textbf{(2.7)}$$

6. Limits of branch flows

$$S_{ij}^2 \leq \mathrm{S_{ijmax}}^2 \qquad \textbf{(2.8)}$$

$$S_{ji}^2 \leq \mathrm{S_{jimax}}^2 \qquad \textbf{(2.9)}$$

Where

P_g, P_d	are the vectors of active power generation and load (MW) respectively
Q_g, Q_d	are the vectors of reactive power generation and load (MVAR) respectively
C_p^2, C_p^1, C_p^0	are the vectors of quadratic (€/MW^2), linear (€/MW) and fixed cost (€) coefficients for active power generation respectively.
V	is the vector of voltages (V)
t	is the vector of transmission control devices such as tap changers, shunts.
$\mathrm{FP_{ij}}, \mathrm{FQ_{ij}}$	are the functions of the active (MW) and reactive power (MVAr) flows from node i to node j respectively
S_{ij}, S_{ji}	are the power flows from node i to j and from j to i respectively (MVA)
MN	is the number of nodes in the system

[1] Since the constraints are applied to all nodes, subscript i can be dropped for the simplification purpose.

The optimisation problem is solved using an advanced Primal Dual Interior Point Method in Non Linear Programming. This method is chosen due to the superior performance of the method compared with other methods [4,5,6].

2.4. Interior Point method

This section describes Non Linear Primal Dual Interior Point optimisation method to solve OPF problem described in the previous section (2.3). This method[2] has been proven to have superiority performance compared with other conventional optimisation methods such as Newton method, sequential quadratic programming, augmented Lagrangian, generalise reduced gradient, projected augmented Lagrangian, and etc especially in the term of the convergence speed, accuracy and the ability to handle inequality constraints.

Formulate the optimisation problem in the previous section in the form below by adding slack variables l and u as the implementation of logarithmic barrier function in order to handle the inequality constraints. l and u will provide the proximity to the variable limits, which will be used to prevent the violation of the limits during the solving process.

$$\begin{aligned}
&\text{minimise. } f(\mathbf{x}) - \mu^k \sum_{i=1}^{n} (\ln l_i + \ln u_i) \\
&\text{subject to. } \mathbf{h}(\mathbf{x}) = \mathbf{0} \\
&\qquad .\mathbf{g}(\mathbf{x}) - \mathbf{l} - \underline{\mathbf{g}} = \mathbf{0} \\
&\qquad .\mathbf{g}(\mathbf{x}) + \mathbf{u} - \overline{\mathbf{g}} = \mathbf{0} \\
&\qquad .(\mathbf{l}, \mathbf{u}) \geq \mathbf{0}
\end{aligned} \tag{2.10}$$

Where: $\mathbf{x} \in \Re^{(n)}$ is a vector of the decision variables,

$f(\mathbf{x})$ is an objective function,

$\mathbf{h}(\mathbf{x}) \equiv [h_1(\mathbf{x}), \cdots, h_m(\mathbf{x})]^T$ is the vector of equality constraints,

$\mathbf{g}(\mathbf{x}) \equiv [g_1(\mathbf{x}), \cdots, g_r(\mathbf{x})]^T$ is the vector of inequality constraints, and

$\overline{\mathbf{g}}$ and $\underline{\mathbf{g}}$ are the vectors of upper and lower bounds respectively.

μ^k is a monotonically decrease barrier parameter along the iteration k-th. A special role of this parameter will be discussed in detail latter.

Use Langrangian function and Karush Kuhn Tucker condition (KKT) to convert the nonlinear optimisation problem into a problem of solving a set of non linear equations. The equations based on the KKT conditions can be expressed as follows.

$$\mathbf{R}_x \equiv \nabla f(\mathbf{x}) - \nabla \mathbf{h}(\mathbf{x})\mathbf{y} - \nabla \mathbf{g}(\mathbf{x})(\mathbf{z} + \mathbf{w}) = \mathbf{0} \tag{2.11}$$

$$\mathbf{R}_y \equiv \mathbf{h}(\mathbf{x}) = \mathbf{0} \tag{2.12}$$

$$\mathbf{R}_z \equiv \mathbf{g}(\mathbf{x}) - \mathbf{l} - \underline{\mathbf{g}} = \mathbf{0} \tag{2.13}$$

$$\mathbf{R}_w \equiv \mathbf{g}(\mathbf{x}) + \mathbf{u} - \overline{\mathbf{g}} = \mathbf{0} \tag{2.14}$$

[2] A relatively large number of papers (more than 100's) describing this method have been published in power system analysis since 1990's.

$$\mathbf{KKT}_{l} \equiv \boldsymbol{LZ}\mathbf{e} - \mu\mathbf{e} = \mathbf{0} \quad \textbf{(2.15)}$$

$$\mathbf{KKT}_{\mathrm{u}} \equiv \boldsymbol{UW}\mathbf{e} + \mu\mathbf{e} = \mathbf{0} \quad \textbf{(2.16)}$$

$$(\mathbf{l}, \mathbf{u}; \mathbf{z}) \geq \mathbf{0}, \mathbf{w} \leq \mathbf{0} \quad \textbf{(2.17)}$$

Where: $\mathbf{R}_{\mathrm{x}}, \mathbf{R}_{\mathrm{y}}, \mathbf{R}_{\mathrm{z}}, \mathbf{R}_{\mathrm{w}}$ denote the optimality conditions associated to the gradients of the Lagrangian function in terms of primal and dual variables,

$\mathbf{KKT}_{\mathrm{l}}$ and $\mathbf{KKT}_{\mathrm{u}}$ denote the complementary conditions,

$(\boldsymbol{L}, \boldsymbol{U}, \boldsymbol{Z}, \boldsymbol{W}) \in \Re^{(r \times r)}$ are diagonal matrices whose elements are **l**, **u**, **z** and **w** respectively,

$\mathbf{y} \in \Re^{(m)}$ and $(\mathbf{z}, \mathbf{w}) \in \Re^{(r)}$ are the Lagrangian multipliers associated with equality and inequality constraints respectively,

$\mathbf{e} = [1, \cdots, 1]^{\mathrm{T}} \in \Re^{(r)}$.

Apply the perturbed Newton method to determine the descent direction for the convergence. The perturbation is required to mitigate or to reduce the possibility of being trapped in the one of variable limits prematurely.

$$\left(\nabla^2\mathbf{h}(\mathbf{x})\mathbf{y} + \nabla^2\mathbf{g}(\mathbf{x})(\mathbf{z} + \mathbf{w}) - \nabla^2 \mathrm{f}(\mathbf{x})\right)\Delta\mathbf{x} + \nabla\mathbf{h}(\mathbf{x})\Delta\mathbf{y} + \nabla\mathbf{g}(\mathbf{x})(\Delta\mathbf{z} + \Delta\mathbf{w}) = \mathbf{R}_{\mathrm{x}0} \quad \textbf{(2.18)}$$

$$\nabla\mathbf{h}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} = -\mathbf{R}_{y0} \quad \textbf{(2.19)}$$

$$\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} - \Delta\mathbf{l} = -\mathbf{R}_{z0} \quad \textbf{(2.20)}$$

$$\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} + \Delta\mathbf{u} = -\mathbf{R}_{\mathrm{w}0} \quad \textbf{(2.21)}$$

$$\boldsymbol{Z}\Delta\mathbf{l} + \boldsymbol{L}\Delta\mathbf{z} = -\mathbf{R}_{\mathrm{l}0}^{\mu} \quad \textbf{(2.22)}$$

$$\boldsymbol{W}\Delta\mathbf{u} + \boldsymbol{U}\Delta\mathbf{w} = -\mathbf{R}_{\mathrm{u}0}^{\mu} \quad \textbf{(2.23)}$$

Where: $\mathbf{R}_{x0}, \mathbf{R}_{y0}, \mathbf{R}_{l0}^{\mu}, \mathbf{R}_{u0}^{\mu}, \mathbf{R}_{z0}$, and $\mathbf{R}_{w0}$ represent the residuals of the perturbed KKT equations.

$\nabla^2\mathbf{h}(\mathbf{x})$ and $\nabla^2\mathbf{g}(\mathbf{x})$ are Hessian matrices of $\mathbf{h}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$.

By substituting $\Delta\mathbf{l}$, $\Delta\mathbf{u}$, $\Delta\mathbf{z}$ and $\Delta\mathbf{w}$ in Equation (2.18) with Equations (2.24)-(2.25), reduced set of system equations in Equations (2.26)-(2.27) can be obtained.

$$\begin{cases} \Delta\mathbf{l} = \nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} + \mathbf{R}_{\mathrm{z}0} \\ \Delta\mathbf{u} = -\left(\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} + \mathbf{R}_{\mathrm{w}0}\right) \end{cases} \quad \textbf{(2.24)}$$

$$\begin{cases} \Delta\mathbf{z} = -\boldsymbol{L}^{-1}\boldsymbol{Z}\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} - \boldsymbol{L}^{-1}\left(\boldsymbol{Z}\mathbf{R}_{\mathrm{z}0} + \mathbf{R}_{\mathrm{l}0}^{\mu}\right) \\ \Delta\mathbf{w} = \boldsymbol{U}^{-1}\boldsymbol{W}\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}}\Delta\mathbf{x} + \boldsymbol{U}^{-1}\left(\boldsymbol{W}\mathbf{R}_{\mathrm{w}0} - \mathbf{R}_{\mathrm{u}0}^{\mu}\right) \end{cases} \quad \textbf{(2.25)}$$

$$\left[\begin{array}{c:c} \boldsymbol{H}(\bullet) & \boldsymbol{J}(\mathbf{x})^{\mathrm{T}} \\ \hdashline \boldsymbol{J}(\mathbf{x}) & \mathbf{0} \end{array}\right]\left[\begin{array}{c} \Delta\mathbf{x} \\ \hdashline \Delta\mathbf{y} \end{array}\right] = -\left[\begin{array}{c} \boldsymbol{\Psi}(\bullet,\mu) \\ \hdashline \mathbf{h}(\mathbf{x}) \end{array}\right]$$

Note: **(2.26)**

Form of linear equation: **A**.**x** = **b**, **A** is the constraint matrix, **x** is the solution vector and **b** is a vector of right hand side (rhs)

where:

$$\begin{cases} \mathbf{H}(\bullet) \equiv \mathbf{H}_{\mathrm{h}} + \mathbf{H}_{\mathrm{g}} = \left(\nabla^2\mathbf{h}(\mathbf{x})\mathbf{y} + \nabla^2\mathbf{g}(\mathbf{x})(\mathbf{z}+\mathbf{w}) - \nabla^2 \mathrm{f}(\mathbf{x})\right) + \nabla\mathbf{g}(\mathbf{x})\mathbf{S}\nabla\mathbf{g}(\mathbf{x})^{\mathrm{T}} \\ \qquad\qquad\qquad \mathbf{S} = \mathbf{U}^{-1}\mathbf{W} - \mathbf{L}^{-1}\mathbf{Z} \\ \mathbf{J}(\mathbf{x}) \equiv \nabla\mathbf{h}(\mathbf{x}) \\ \boldsymbol{\Psi}(\bullet,\mu) \equiv -\mathbf{R}_{\mathrm{x0}} - \nabla\mathbf{g}(\mathbf{x})\left(\mathbf{U}^{-1}\left(\mathbf{W}\mathbf{R}_{\mathrm{w0}} - \mathbf{R}_{\mathrm{u0}}^{\mu}\right) - \mathbf{L}^{-1}\left(\mathbf{Z}\mathbf{R}_{\mathrm{z0}} + \mathbf{R}_{\mathrm{l0}}^{\mu}\right)\right) \\ \qquad = \nabla\mathbf{h}(\mathbf{x})\mathbf{y} - \nabla\mathrm{f}(\mathbf{x}) + \nabla\mathbf{g}(\mathbf{x})\left(\mathbf{U}^{-1}\mathbf{W}\mathbf{R}_{\mathrm{w0}} - \mathbf{L}^{-1}\mathbf{Z}\mathbf{R}_{\mathrm{z0}} - \mu\left(\mathbf{U}^{-1} - \mathbf{L}^{-1}\right)\right) \end{cases} \quad \mathbf{(2.27)}$$

The PDIPM algorithm [7], which will be used to solve the problem, can be summarised as follows:

Step 0: Initialisation: Set k=1, $K_{\max}$=200, centring parameter $\sigma \in (0,1]$, and convergence criteria, calculate r = number of inequality constraints, choose $\mathbf{u},\mathbf{l} > \mathbf{0}$ and $\mathbf{z} > \mathbf{0}, \mathbf{w} < \mathbf{0}, \mathbf{y} = \mathbf{0}$, where k, $K_{\max}$ are the iteration counter and its maximum respectively.

WHILE (k< $K_{\max}$) DO:

Step 1: *Compute the complementary gap:*

$$\mathrm{C}_{\mathrm{gap}} \equiv \sum_{\mathrm{i}=1}^{\mathrm{r}} \mathrm{l}_{\mathrm{i}}\mathrm{z}_{\mathrm{i}} - \mathrm{u}_{\mathrm{i}}\mathrm{w}_{\mathrm{i}} \quad \mathbf{(2.28)}$$

If the convergence criteria, which comprises the maximum of active and reactive mismatched and complementary gap, is satisfied, then the current result is considered as the optimal solution and the iterative process can be stopped.

Step 2: Compute the perturbed factor

$$\mu \equiv \sigma\frac{\mathrm{C}_{\mathrm{gap}}}{2\mathrm{r}} \quad \mathbf{(2.29)}$$

Step 3: Solve Equation (2.26) for $[\Delta\mathbf{x}, \Delta\mathbf{y}]$

Step 4: Given $\Delta\mathbf{x}$, calculate $\Delta\mathbf{l}, \Delta\mathbf{u}$ and $\Delta\mathbf{z}, \Delta\mathbf{w}$ using Equations (2.24) and (2.25) respectively.

Step 5: Perform the ratio test to determine the maximum step length in the primal and dual space:

$$\mathrm{step_P} = 0.9995\min\left\{\min_{i}\left(\frac{-l_i}{\Delta l_i}\ \text{when}\ \Delta l_i < 0; \frac{-u_i}{\Delta u_i}\ \text{when}\ \Delta u_i < 0\right), 1\right\} \tag{2.30}$$

$$\mathrm{step_D} = 0.9995\min\left\{\min_{i}\left(\frac{-z_i}{\Delta z_i}\ \text{when}\ \Delta z_i < 0; \frac{-w_i}{\Delta w_i}\ \text{when}\ \Delta w_i > 0\right), 1\right\} \tag{2.31}$$

Note that direct update of the variables using the increment found in steps 3 and 4, cannot be used as it may result in a violation of the constraints. Consider a variable *d* such that $d \geq 0$. Suppose that the increment at the *k-th* iteration is negative, $\Delta d^{k} \leq 0$. In order to enforce non-negativity of the variable, $d^{(k+1)} = d^{(k)} + step\Delta d^{(k)} \geq 0$, the parameter *step* must satisfy the following condition, $step \leq \dfrac{-d^{(k)}}{\Delta d^{(k)}}$. In order to ensure the numerical stability of the algorithm, *step* is calculated as $step = 0.9995\dfrac{-d^{(k)}}{\Delta d^{(k)}}$

Step 6: Update the primal and dual variables by:

$$\begin{bmatrix}\mathbf{x}\\ \mathbf{l}\\ \mathbf{u}\end{bmatrix}^{k+1} = \begin{bmatrix}\mathbf{x}\\ \mathbf{l}\\ \mathbf{u}\end{bmatrix}^{k} + \mathrm{step_p}\begin{bmatrix}\Delta\mathbf{x}\\ \Delta\mathbf{l}\\ \Delta\mathbf{u}\end{bmatrix}^{k} ; \begin{bmatrix}\mathbf{y}\\ \mathbf{z}\\ \mathbf{w}\end{bmatrix}^{k+1} = \begin{bmatrix}\mathbf{y}\\ \mathbf{z}\\ \mathbf{w}\end{bmatrix}^{k} + \mathrm{step_D}\begin{bmatrix}\Delta\mathbf{y}\\ \Delta\mathbf{z}\\ \Delta\mathbf{w}\end{bmatrix}^{k} \tag{2.32}$$

Step 7: Increase index k by 1

END DO

Step 8: Print “Computation does not converge”.

Step 8 indicates the non-convergence of the iterative process. The problem needs to be investigated further to find the causes of the convergence problem.

3. Evaluation of the proposed approach in microgrids

This section describes a number of studies that were conducted to illustrate the implementation of the proposed losses allocation and the related charges into a microgrid connected to a public electricity system. The studies were performed on the developed "Study-Case LV Network" [3] test system. The first case study illustrates the impact of micro sources on losses. The second case study demonstrates the allocation of cost of losses directly in nodal charges for each bus for 24 hours daily operation. The results were observed and some key characteristics were described. The third case study shows that there is a mismatch between generation payment and demand charges. This problem should be resolved by using nodal charges adjustment to maintain net zero mismatch. It is assumed that revenue for network companies is obtained from network charges and not from the energy market. The method to adjust nodal charges is presented and described in the third case study.

3.1. Data for LV test system

The Low Voltage (LV) network single line diagram is presented in Figure 3-1. The system consists of three feeders: a residential feeder (leftmost), an industrial feeder (middle) and a commercial feeder (rightmost). Each feeder has different load characteristics.

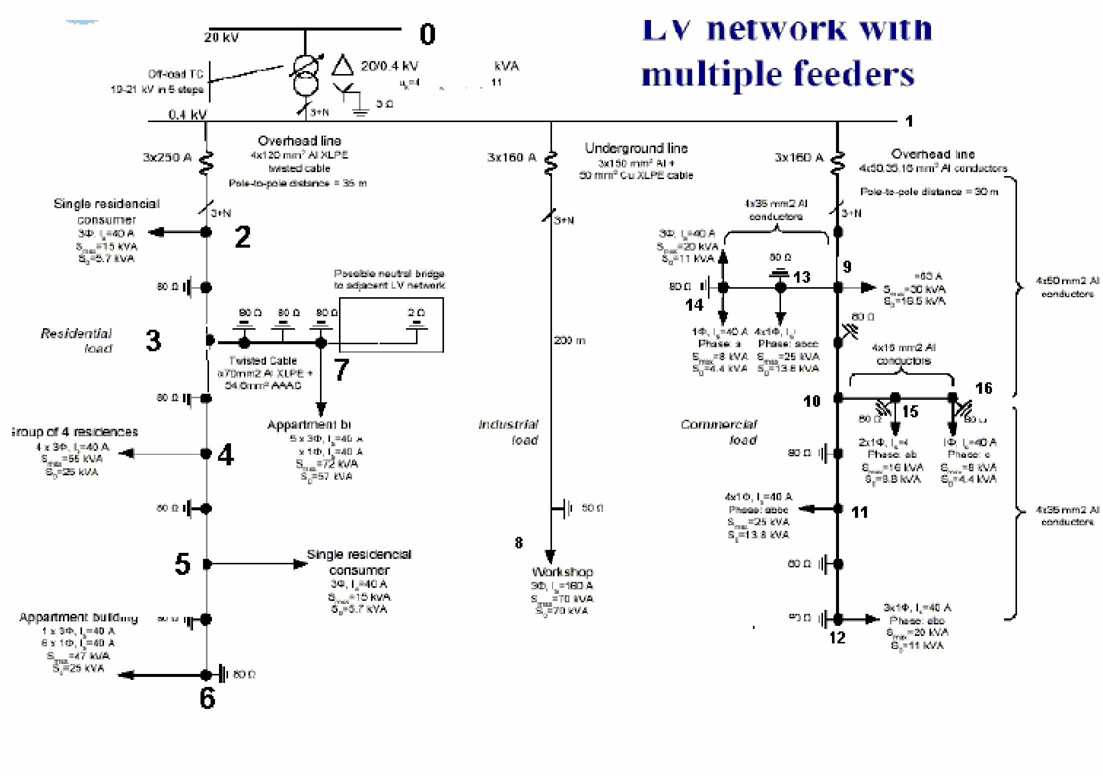


Figure 3-1 Single line diagram of the LV network study case.

[3] System characteristics are detailed in the document Study-Case LV Network.pdf by Stavros Papathanassiou (available in http://microgrids.power.ece.ntua.gr/documents/Study-Case%20LV-Network.pdf)

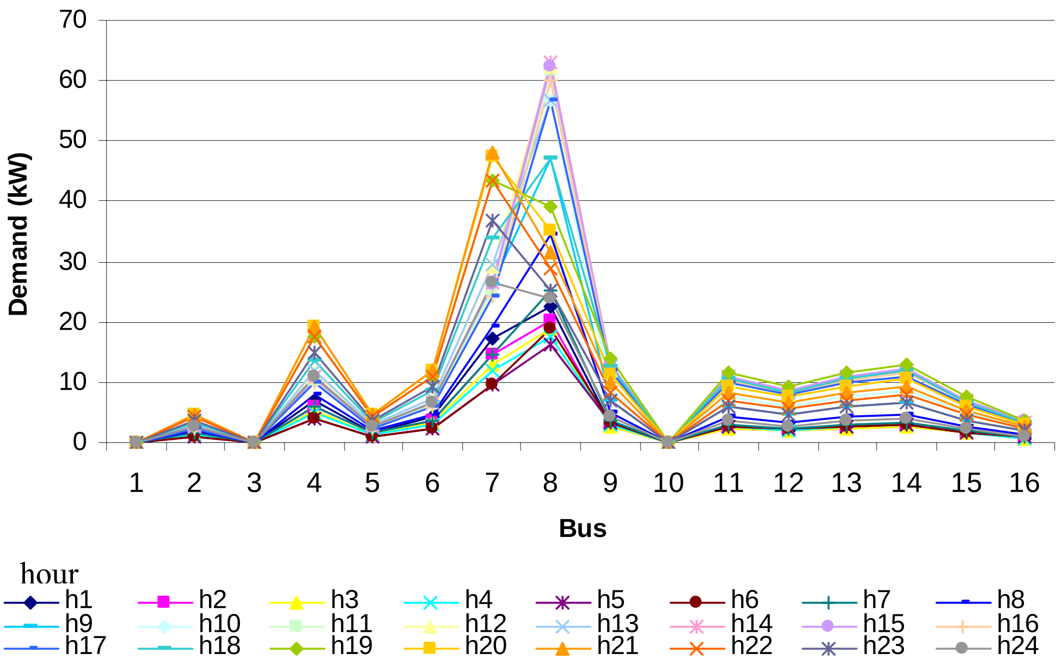


Figure 3-2 24 hour demand profiles

Figure 3-2 shows 24 hours demand profiles obtained by combining the load time series from each demand customer in the microgrid. The minimum total demand is 51 MW at hour 5 and the maximum demand is 191 MW at hour 19. Power factor of loads is assumed to be 0.85 lagging.

For the purpose of the study, the number and total capacity of micro sources in the system was significantly increased from 88 kW to 150 kW. The installed maximum capacity and bids data for each generator are presented in Table 3-1. The location of each micro source in the test system is shown in Figure 3-3.

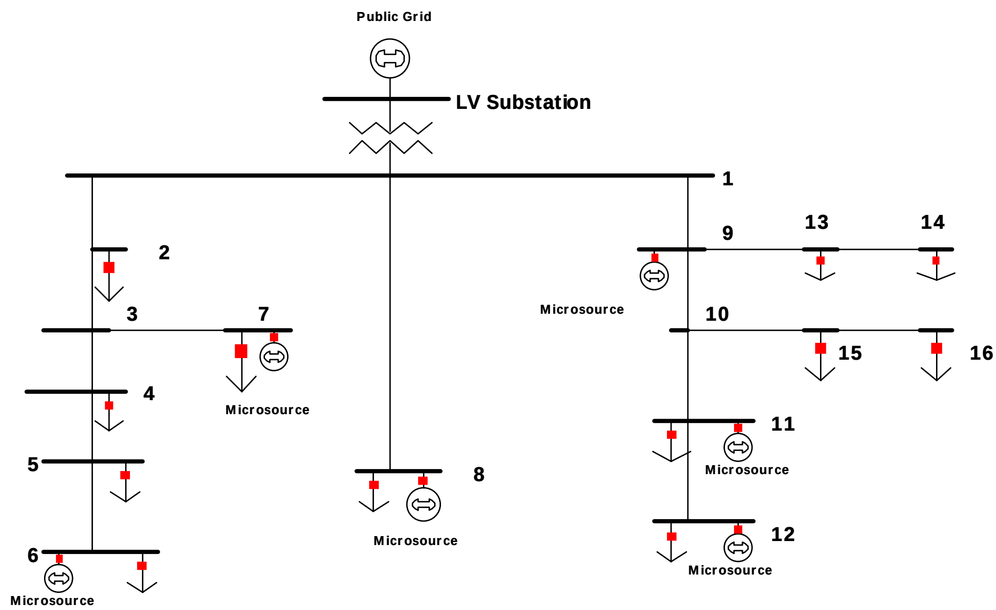


Figure 3-3 The LV microgrid test system

Table 3-1 Generation installed capacity and generation bids data

Location (bus)	Installed maximum capacity (kW)	Generation bids (€ct/kWh)
6	30	3.04
7	30	5.16
8	50	8.00
9	10	10.00
11	15	11.00
12	15	15.00

The quadratic cost coefficients for all generation are very small (0.01 €ct/kWh^2) and the cost of reactive power is set to almost zero (0.01 €ct/kVAr). Hence, the cost functions of generators are approximately linear. It is also assumed that for case study 1-3, the cost of importing power from public electricity system is more expensive than the cost of generating power locally. It is important to note that the figures in Table 3-1 were developed for illustration purpose only.

3.2. Case study I: Impact of micro sources on losses

The impact of micro sources on the LV test system losses is demonstrated in Figure 3-4. The figure shows the variation of losses for a system without and with micro sources. The first (left) y-axis denotes kW load and the second y-axis denotes the losses in the microgrid in the percentage of total demand at the corresponding period.

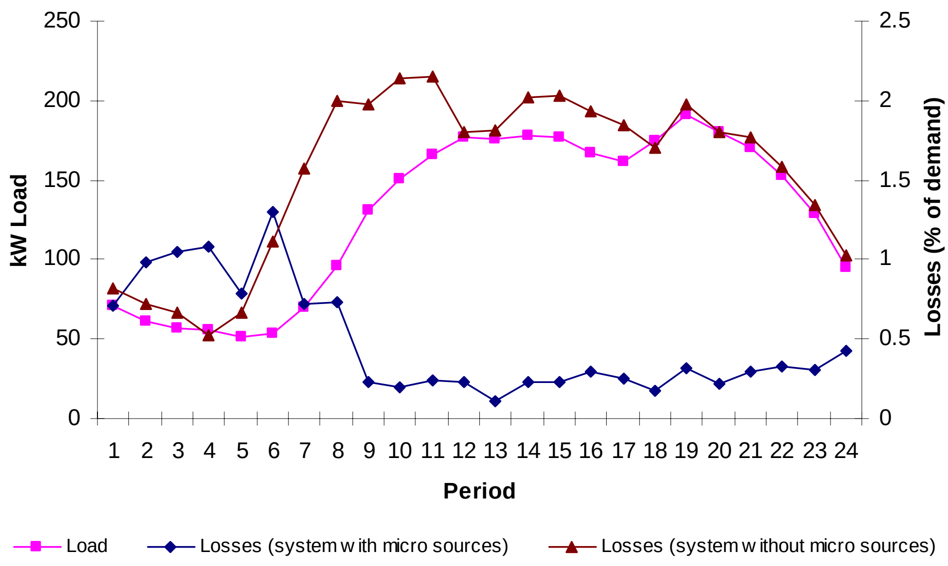


Figure 3-4 Micro sources contribution on losses reduction

This case study demonstrates the losses reduction related benefits of micro generation located close to the load. Without local sources, losses in the LV test system will be between the range of 0.29% – 3.77% of total demand with the average losses of 1.71% of total demand. With local generation, the average losses in the LV system can be reduced up to 0.35% of total demand. The reduction of 1.36% is relatively significant. Moreover, the reduction has not included the reduction of losses up stream.

It is interesting to note that the characteristics of losses profile with and without micro generation are different in this case. For the system with micro generation, the peak losses occurred in the off peak demand periods and the percentage of losses became smaller in the peak demand conditions. The result is expected since merit generation in the microgrid located at the end of the residential feeder and therefore the network impedance seen by the merit generator to supply loads is relatively large. In the peak load, all local micro sources were fully engaged and generation was distributed across the microgrid. Without local sources, it was expected that the losses would increase following the increase in demand as shown in Figure 3-4.

3.3. Case study II: Allocation of cost of losses

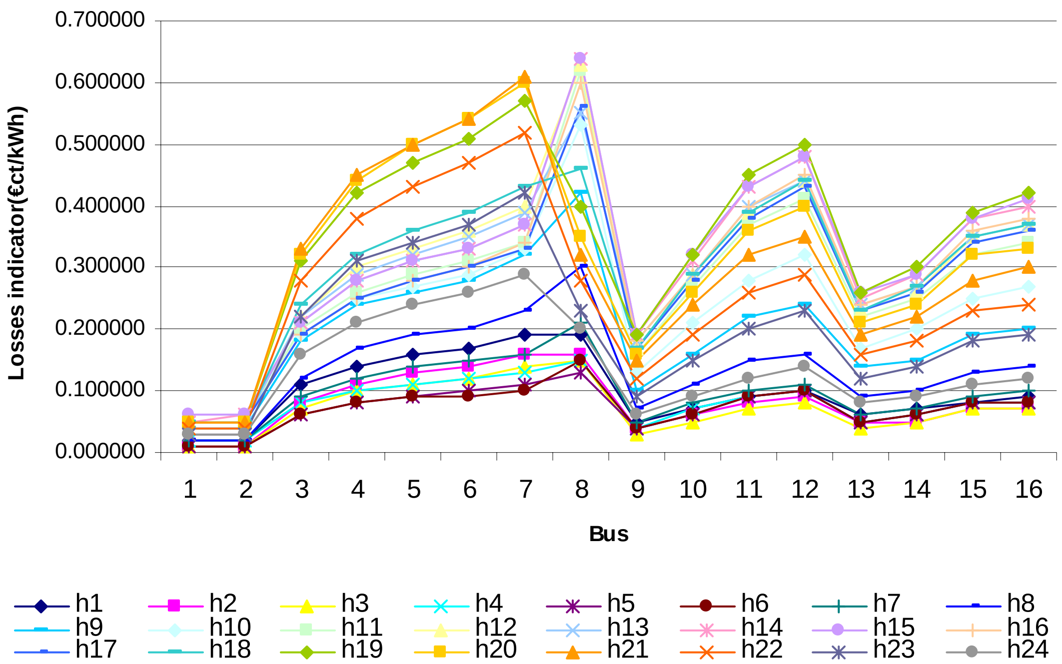


Figure 3-5 Micro sources contribution on losses reduction

Figure 3-5 shows the economic indicators that measure the contribution of losses to operation costs from each demand customer in the microgrid without micro sources. The indicators were obtained by subtracting the nodal charges for each node with the nodal charges at the marginal generator node. In this case, the marginal generator node is the LV substation. A positive or negative indicator associated to a node means that losses and the cost of losses will increase or reduce respectively for additional power taken from the node. Although it is unlikely, the indicator can also become zero. This means that the incremental of load will not affect system losses. The losses indicators for generation have the same magnitude but opposite sign.

Several key points observed from this study are listed as follows:

1. As expected, customers located further from the LV substation contribute to losses higher than customers located closer to the substation. Therefore, it is fair to charge customers at the end of the feeder higher than customers at the substation. On the other hand, future micro sources located at the end of

the feeder have higher value in term of reduction of losses related benefits compared with micro sources at the substation.

2. Since the source of supply was one (the public electricity system), the profiles of losses indicators are consistently following the loads.

This study also investigated the pattern of losses indicators in the microgrid with merit micro sources. Figure 3-6 shows the variations of losses indicators for 24 hours for each node. With a number of micro sources distributed across the microgrid, it is now possible to have positive and negative losses indicators as shown in Figure 3-6. The losses indicators computed for minimum demand condition at hour 5 (Figure 3-7 a) has different profile with the ones computed for maximum demand condition at hour 19 (Figure 3-7 b). The larger magnitude of losses indicators in the peak demand periods indicates that the cost of losses in these periods is higher due to the use of more expensive generating units to supply demand.

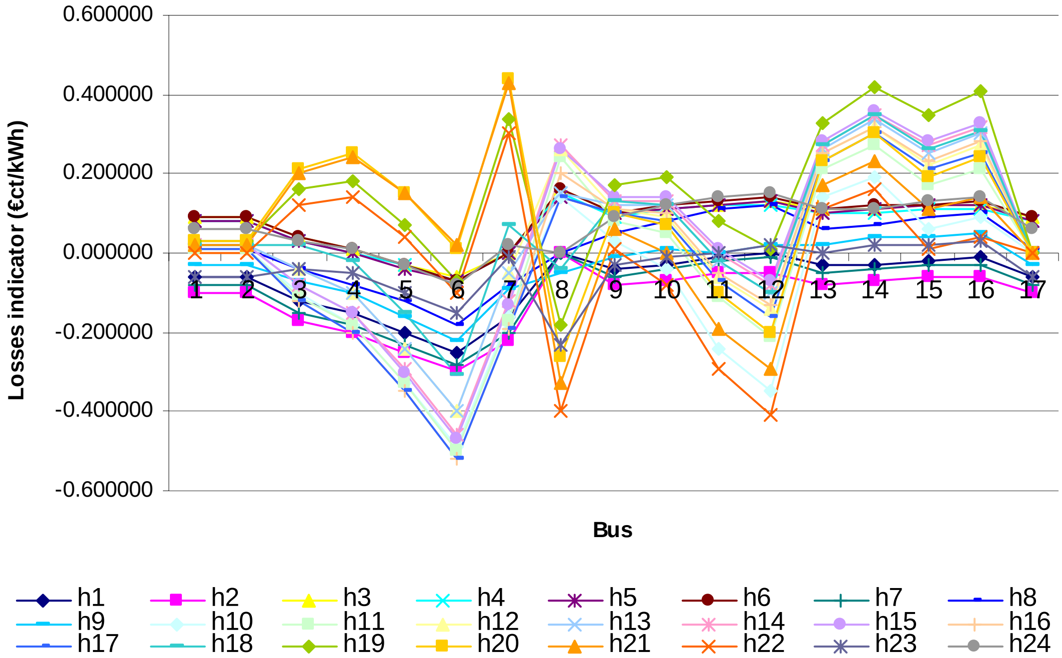
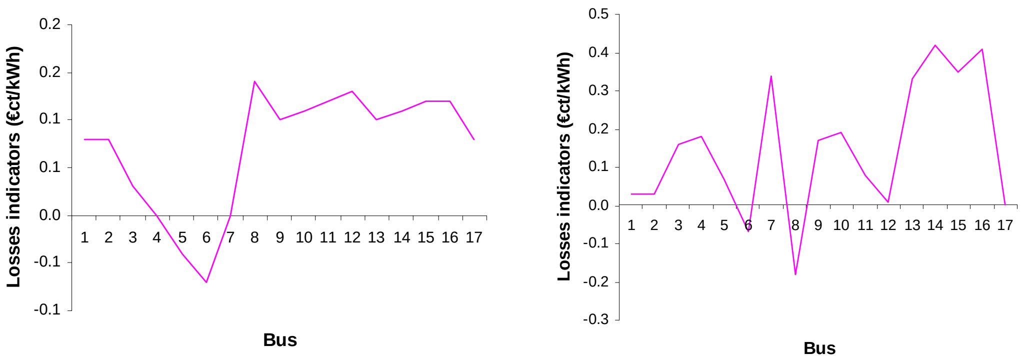


Figure 3-6 Micro sources contribution on losses reduction



(a) minimum demand (hour 5) (b) maximum demand (hour 19)

Figure 3-7 Micro sources contribution on losses reduction

Several key points observed from this study are listed as follows:

1. The profiles of losses indicators dynamically vary following the generation output patterns and load profiles. It shows the time of use and location specific aspects of the losses indicators. For example, the polarity and magnitude of losses indicators for bus 1 and some other buses change in time. The indicators will then provide economic incentives to demand to manage more efficiently their consumption for example by shifting the use of electricity in peak demand to off peak demand. This will increase the efficiency of system as whole.

2. Due to the fluctuation of losses indicators, it becomes more challenging to determine the next generation investment location in the microgrid, which has high value in term of reducing losses. By inspection, it can be seen that new micro sources are required in the commercial feeders (bus 13-16) since the losses indicators for exit from these buses is positive. In contrast, installation of new generation at bus 6 is not desirable since it will increase losses in the system as shown in Figure 3-6.

3. For demand customers, the location close to LV substation or to merit order generators is the location where the charges due to losses are small.

4. It is interesting to note that nodal charges at merit generator nodes are slightly smaller than the nodal charges at the marginal generator nodes. Consequently, the use of nodal pricing to determine the payment for generators will reduce the amount of generator revenue compared with if the generator payment is determined by system marginal price. The rational behind this phenomenon is that the power transmitted by merit generators incurs losses. Therefore, increasing generation capacity in those nodes are not encouraged in the context of reducing losses.

3.4. Case study III: Reconciliation

In this case study, a revenue reconciliation method is proposed to maintain the equality of total generation payment and total demand charges. With the proposed approach, the total generation payment (Table 3-2) is slightly smaller to the total demand charges (Table 3-3) due to the losses. Hence, a form of revenue reconciliation is needed.

Table 3-2 Generation payment at hour 9

Location (bus)	Generation (kW)	Tariff (€ct/kWh)	Revenue (€ct/h)
6	30.0	11.01	330.3
7	30.0	11.14	334.2
8	50.0	11.18	559
9	10.0	11.22	112.2
11	11.6	11.23	130.268
12	0.0	11.25	0
			1465.968

Table 3-3 Demand charges at hour 9

Location (bus)	Demand (kW)	Tariff (€ct/kWh)	Charges (€ct/h)
1	0.0	11.20	0.000
2	2.6	11.20	29.12
3	0.0	11.16	0.000
4	10.6	11.13	117.978
5	2.6	11.07	28.782
6	6.5	11.01	71.565
7	26.2	11.14	291.868
8	47.2	11.18	527.696
9	7.0	11.22	78.54
10	0.0	11.24	0.000
11	5.9	11.23	66.257
12	4.7	11.25	52.875
13	5.9	11.25	66.375
14	6.5	11.27	73.255
15	3.7	11.27	41.699
16	1.9	11.28	21.432
			1467.442

In this study, a simple revenue reconciliation method is proposed. Assuming that generators have fully recovered their operational costs given that the nodal charges for the merit generators are at least equal to their bids, only demand charges then need to be adjusted such that the total demand charges is equal to the total generation payment. In order to maintain the competitiveness between network customers, the difference between nodal charges across the system is maintained. The selected revenue reconciliation method uses a shifting factor to reduce the nodal charges at all nodes equally. The factor can be obtained easily by dividing the difference between the total demand charges (€ct 1467.442/h) and total generation payment (€ct 1465.968/h) with the total demand (131.3 kW). The result is €ct 0.011226/kWh. New demand charges are presented in Table 3-4. It is evident that the total demand charges are now equal to the total generation payment.

Table 3-4 Demand charges at hour 9

Location (bus)	Demand (kW)	Tariff (€ct/kWh)	Charges (€ct/h)
1	0.0	11.19	0.00
2	2.6	11.19	29.09
3	0.0	11.15	0.00
4	10.6	11.12	117.86
5	2.6	11.06	28.75
6	6.5	11.00	71.49
7	26.2	11.13	291.57
8	47.2	11.17	527.17
9	7.0	11.21	78.46
10	0.0	11.23	0.00
11	5.9	11.22	66.19
12	4.7	11.24	52.82
13	5.9	11.24	66.31
14	6.5	11.26	73.18
15	3.7	11.26	41.66
16	1.9	11.27	21.41
			1465.96

4. Summary

This report has presented an approach to allocate the cost of losses to nodal charges using a Non Linear Interior Point Method based AC OPF. This approach resolves the deficiencies of some other methods and can be integrated directly in the operation of energy market. The ability of the method to be used in the real time pricing also omits the requirement of ex ante computation for allocation of losses and the adjustment needed afterward.

The economic signals to stimulate efficiency in microgrids system operation and investment are provided in the approach. Losses indicators are derived from the nodal charges obtained from the result of solving OPF problem. The problem formulation and the interior point algorithm have been described in this report. It is important to note that the OPF problem does not minimise losses directly but it minimises the total operation costs in the system including cost of losses. Therefore, the nodal charges as the resultant from the OPF problem reflects the optimal charges to network customers, both generation and demand.

A number of case studies have been conducted to illustrate the features of the proposed approach in the developed microgrid LV test system. The results demonstrate that the losses indicators provide a measurement to network users about their temporal and spatial contribution on the incremental cost of losses in the system. The cost reflective nodal charges taking into account losses are provided to influence operation and investment strategic decisions of individual customers to improve the overall system efficiency.

In the end of this report, a relatively simple revenue reconciliation method was demonstrated. The method preserves the competitiveness of network customers by maintaining the difference between tariffs across the system. This aspect is crucial especially in competitive market environment.

5. References

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Report

Loss Allocation in Low Voltage Networks with Dispersed Generation

December 2003

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1. Introduction

One of the basic objectives of the re-deregulation of electricity markets is the promotion of competition same as in the production in the sector of sales, considering that it will lead to lower electricity prices and to an improvement of quality of provided services. Essential pre-condition for competition to develop is an open and without discriminations access to the transmission and distribution networks as well as to the low voltage networks (LV) by all the players of energy market. The idea for an open and without discriminations access is a network pricing issue. The network costs comprise mainly three components, the investment costs, the congestion costs and the operating costs part of which and the most significant is the cost of losses. The allocation of this cost to each individual generation and load is in essence the allocation of responsibility for the system losses.

Today's competitive energy markets are faced with environment, higher standards of service reliability and economic issues and the need for more competitive nonconventional suppliers of electricity. Dispersed Generation (DG) improves the energy quality and under some conditions decreases the network's operational cost.

Thus arise the need of new tools that provide better estimations of the final compensations and the appropriate signals to users of the network to motivate economically efficient operating decisions.

The loss allocation method applied uses the concept of marginal losses to derive marginal prices. The method provides loss allocation factors for both active and reactive power enabling the contribution of active and reactive power consumption/generation to system losses to be quantified. Furthermore, the factors can be positive or negative reflecting the user's impact on losses, which is essential in addressing the impact of counter flows, preventing thus temporal and spatial cross-subsidies. Also a mechanism for neutralising the impact of choice of reference node on the magnitude and the polarity of loss allocation factors by apportioning total losses equally between generators (including the reference node) and loads is applied.

The method is applied to the LV test network, available in
http://microgrids.power.ece.ntua.gr/documents/Study-Case%20LV-Network.pdf

2. Loss Allocation Method

A. Marginal Loss Coefficients

According to the economic theory the marginal losses reflect the Short-Term Marginal Costs (STMC) and therefore achieve short-term economic efficiency [5], [10]. The marginal loss coefficients (MLC's) are sensitivity factors measuring the change of total active losses L when a marginal change in consumption/generation of active P_i and reactive power Q_i occurs at each node i in the network. Then:

$$\tilde{\alpha}_{P_i} = \frac{\partial L}{\partial P_i}, \quad \tilde{\alpha}_{Q_i} = \frac{\partial L}{\partial Q_i}, \tag{1}$$

where $\tilde{\alpha}_{P_i}$ and $\tilde{\alpha}_{Q_i}$ are the active and reactive MLC's respectively. For the voltage control nodes (PV) there are no loss related charges for the reactive power they are inject in the system. There are no loss related charges for the reference bus as well, such as he inject/absorb power to keep the system in power balance after changes in injections in other nodes. This is expressed by:

$$\frac{\partial L}{\partial Q_i} = 0 \quad , i \in \{PV\} \tag{2}$$

$$\frac{\partial L}{\partial P_r} = \frac{\partial L}{\partial Q_r} = 0 \;\; , r \equiv reference\, bus \tag{3}$$

It is obvious that the choice of reference node is crucial for magnitude and polarity of the calculated MLC's. The neutralization of this will be presented in next paragraph.

The calculation of MLC's is based on a solved power flow in a particular operating point of the system. The voltages and angles used as intermediate state variables as there is no explicit relationship between losses and power injections. Applying the standard chain rule, the following system of linear equations gives the MLC's

$$\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial Q}{\partial \theta} \\ \frac{\partial P}{\partial V} & \frac{\partial Q}{\partial V} \end{bmatrix} \cdot \begin{bmatrix} \tilde{\alpha}_P \\ \tilde{\alpha}_Q \end{bmatrix} = \begin{bmatrix} \frac{\partial L}{\partial \theta} \\ \frac{\partial L}{\partial V} \end{bmatrix} \Rightarrow \begin{bmatrix} \tilde{\alpha}_P \\ \tilde{\alpha}_Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial Q}{\partial \theta} \\ \frac{\partial P}{\partial V} & \frac{\partial Q}{\partial V} \end{bmatrix}^{-1} \cdot \begin{bmatrix} \frac{\partial L}{\partial \theta} \\ \frac{\partial L}{\partial V} \end{bmatrix} \tag{4}$$

where the first term is the transposed Jacobian matrix $\left[J^T \right]$.

The approximately quadratic relationship between losses and power flows is responsible for the twice amount of losses calculated applying the MLC's to the following equation:

$$\sum_{i=1}^{N} \tilde{\alpha}_{P_i} \cdot \left(P_{g_i} - P_{l_i} \right) + \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \tilde{\alpha}_{Q_i} \cdot \left(Q_{g_i} - Q_{l_i} \right) \approx 2 \cdot L \tag{5}$$

where P_{g_i} and P_{l_i} are the active power generation and demand at node i respectively and Q_{g_i}, Q_{l_i} are the reactive power generation and demand at node i respectively.

B. *Constant Multiplier Reconciliation Factor*

While the losses calculated are equal to approximately twice the actual amount of losses, arises the need of MLC's reconciliation so as to yield the exact amount of revenue that is required. The reconciliation method where proposed is to apply a constant multiplier in the order of 50%. In this method the MLC's are simply scaled down s as to yield the amount of the required revenue. To obtain the reconciled vector of MLC's a constant multiplier reconciliation factor κ_M is applied:

$$\kappa_M = \frac{L}{\sum_{i=1}^{N} \tilde{\alpha}_{P_i} \cdot \left(P_{g_i} - P_{l_i} \right) + \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \tilde{\alpha}_{Q_i} \cdot \left(Q_{g_i} - Q_{l_i} \right)} \tag{6}$$

where L obtained from the power flow calculation. The value of κ_M is approximately equal to 0.5 (i.e. $\kappa_M \approx 0.5$) and the vectors of MLC's, $\tilde{\alpha}_{PM}$ and $\tilde{\alpha}_{QM}$ which are reconciled by the constant scaling factor κ_M, are then calculated as follows:

$$\tilde{\alpha}_{PM} = \kappa_M \cdot \tilde{\alpha}_P \quad \text{and} \quad \tilde{\alpha}_{QM} = \kappa_M \cdot \tilde{\alpha}_Q \tag{7}$$

Reconciled MLC's enable the allocation of the total system active power losses to individual users such that:

$$\sum_{i=1}^{N} \tilde{\alpha}_{PM_i} \cdot \left(P_{g_i} - P_{l_i} \right) + \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \tilde{\alpha}_{QM_i} \cdot \left(Q_{g_i} - Q_{l_i} \right) \approx L \tag{8}$$

Reconciliation by constant multiplier factor has the tendency to weaken economic signals by diminishing price differentials between nodes.

C. Neutralising of Choice of Reference Node

The assumption that the MLC's at reference node are zero, as mentioned above, has as consequence change of this node to lead to a completely different set of MLC's in terms of magnitude and polarity. It is important for the method to be seen to be consistent by yielding consistent values of MLC's irrespective of choice of reference node. This can be obtained by maintaining a constant ratio of contribution to total losses by generators (or loads). By shifting both active and reactive power loss allocation related factors by constant factors δ_P and δ_Q respectively a given generator loss contribution ratio μ can be achieved. The values of δ_P and δ_Q required to achieve μ per unit of losses being assigned to generators can calculated respectively as follows:

$$\delta_P = \left(\sum_{i=1}^{N} \tilde{\alpha}_{P_i} \cdot P_{g_i} + \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \tilde{\alpha}_{Q_i} \cdot Q_{g_i} - \mu \cdot L \right) \Bigg/ \left(\left(\sum_{i=1}^{N} \left(P_{g_i} - P_{l_i} \right) \Bigg/ \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \left(Q_{g_i} - Q_{l_i} \right) \right) \cdot \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} Q_{g_i} - \sum_{i=1}^{N} P_{g_i} \right) \tag{9}$$

$$\delta_Q = - \left(\sum_{i=1}^{N} \left(P_{g_i} - P_{l_i} \right) \Bigg/ \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \left(Q_{g_i} - Q_{l_i} \right) \right) \cdot \delta_P \tag{10}$$

where P_{g_i} and P_{l_i} are the active power generation and demand at node i respectively and Q_{g_i}, Q_{l_i} are the reactive power generation and demand at node i respectively.

For equal overall apportionment of losses between generation and losses a value of μ equal to 0.5 should be used. The finally allocation of the total system active power losses to individual users, irrespective of choice of reference node is given from the following equation:

$$\sum_{i=1}^{N} \left(\tilde{\alpha}_{PM_i} + \delta_P \right) \cdot \left(P_{g_i} - P_{l_i} \right) + \sum_{\substack{i=1 \\ i \notin PV,ref}}^{N} \left(\tilde{\alpha}_{QM_i} + \delta_Q \right) \cdot \left(Q_{g_i} - Q_{l_i} \right) \approx L \tag{11}$$

The Payment Factors (PF) for the active and reactive injections to the network for node i are:

$$PF_{P_{gi}} = \left(\tilde{\alpha}_{PM_I} + \delta_P \right) \cdot P_{g_i} \tag{12}$$

$$PF_{Q_{gi}} = \left(\tilde{\alpha}_{QM_I} + \delta_Q \right) \cdot Q_{g_i} \tag{13}$$

The revenue for each generator can be calculated as follows:

$$REV_{P_{gi}} = C \cdot P_{g_i} \cdot \left(1 - \left(\tilde{\alpha}_{PM_I} + \delta_P \right) \right) \tag{14}$$

where $C = € / kWh$ and P_g in MW. The difference of the revenues assessed before and after the MLC's appliance gives the revenue percentage change.

$$RPC_{P_{gi}} = 100 \cdot \left\lfloor REV_{P_{gi}} - C \cdot P_{g_i} \right\rfloor \Big/ C \cdot P_{g_i} \tag{15}$$

3. Case Study

A. LV network

The method is applied to the LV test network shown in the following fig.1. The DG is located only in the first (Residential feeder) while the next two are an industrial and a commercial load feeder respectively.

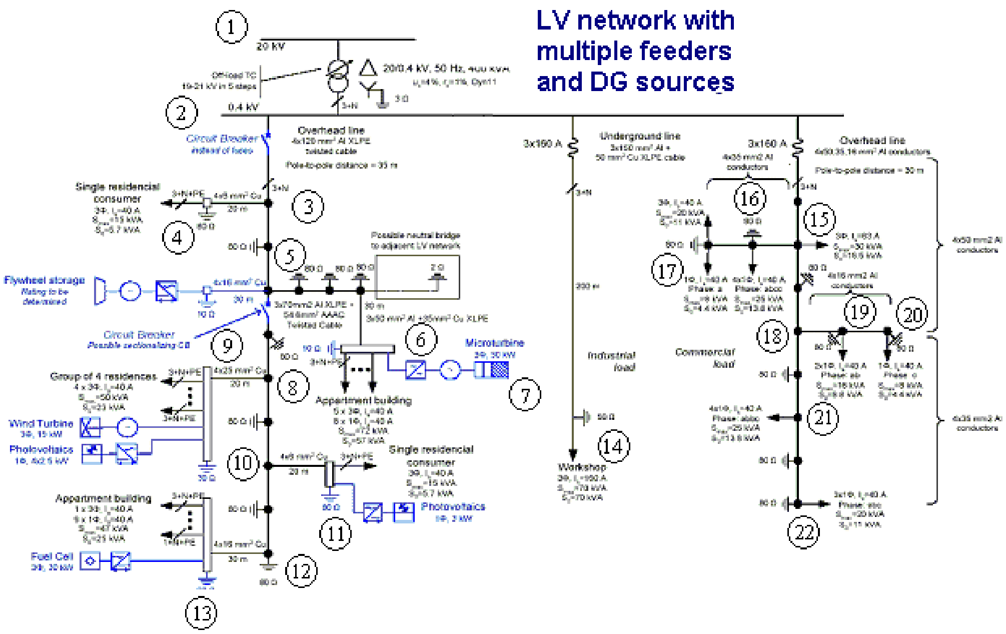


Fig. 1. LV network used for simulations

Table 1. LV lines characteristics

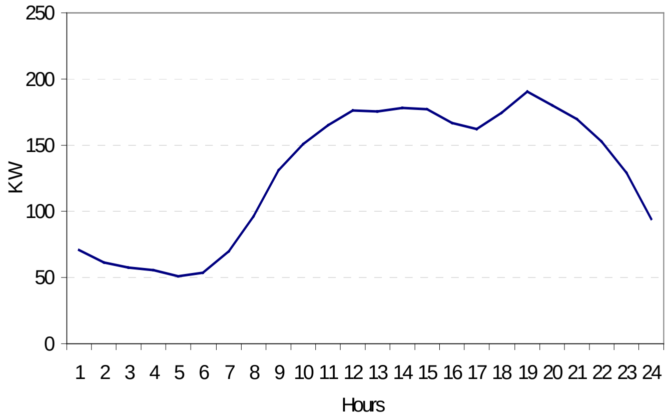
	Line type	R (Ω/km)	X (Ω/km)	R_n (Ω/km)
1	Overhead - Twisted cable 4x120 mm^2 Al	0.284	0.083	0.284
2	Overhead - Twisted cable 3x70 mm^2 Al + 54.6 mm^2 AAAC	0.497	0.100	0.630
3	Overhead – Conductors 4x50 mm^2 Al	0.397	0.279	
4	Overhead – Conductors 4x35 mm^2 Al	0.574	0.294	
5	Overhead – Conductors 4x16 mm^2 Al	1.218	0.318	
6	Underground – XLPE cable 3x150 mm^2 Al + 50 mm^2 Cu	0.264	0.071	0.387
7	Connection - Cable 4x6 mm^2 Cu	3.690	0.094	
8	Connection - Cable 4x16 mm^2 Cu	1.380	0.082	
9	Connection - Cable 4x25 mm^2 Cu	0.871	0.081	
10	Connection - Cable 3x50 mm^2 Al + 35 mm^2 Cu (XLPE)	0.822	0.077	0.524
11	Connection - Cable 3x95 mm^2 Al + 35 mm^2 Cu (XLPE)	0.410	0.071	0.524

NOTES:

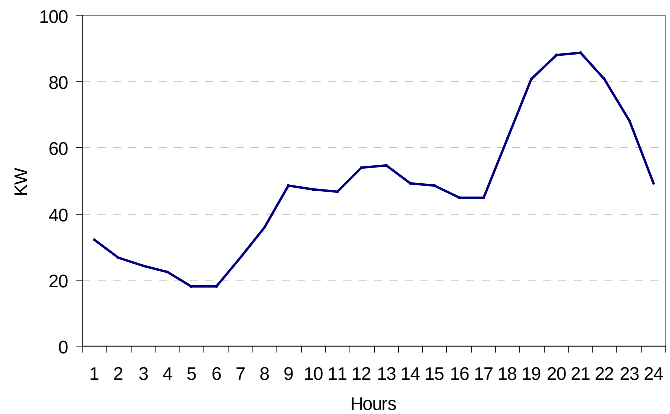
- For types 3-5 the copper equivalent cross-sections are given. Actual Al conductor cross-sections are 27, 57 and 82 mm2, respectively.
- Ohmic resistances for types 1-5 are calculated at 50 oC. For types 6, 10 and 11, at 90 oC for phase conductors and 20 oC for the neutral. For types 7-9 at 70 oC (all conductors).

B. *Loads*

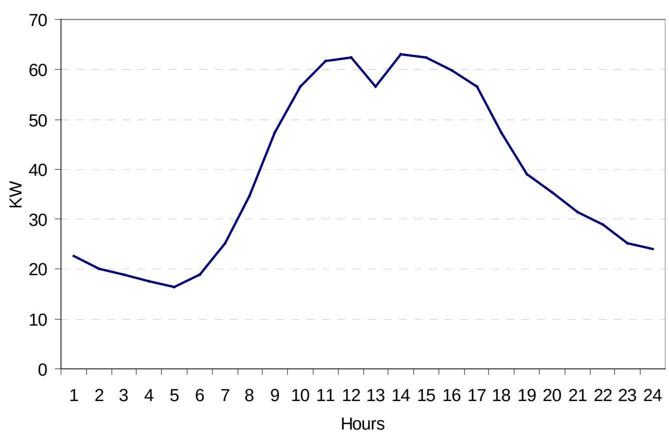
The total load time series and the residential, industrial, and commercial loads series are given in fig 2. For simplicity reasons and without loss of generality, each type of load has been allocated to the relevant nodes according to their average apparent power installed consumption.



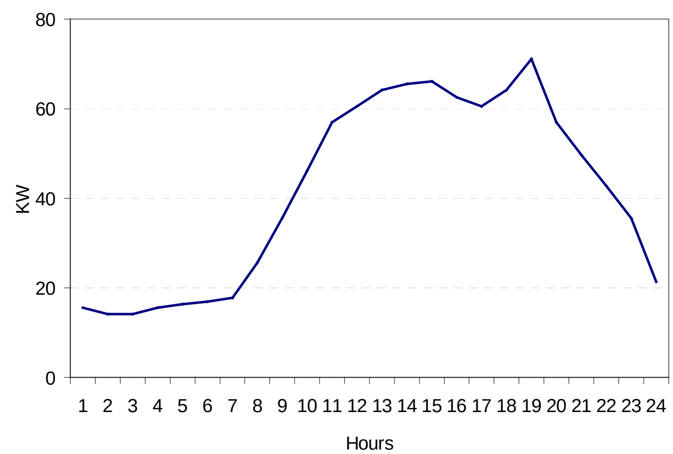
Total Active Load



Residential Active Load



Industrial Active Load



Commercial Active Load

Fig. 2. Load profiles

C. Results

The first results concern the active power losses without power production from the DG's and with power production from them. As it was expected the DG power production reduces the losses of the LV network.

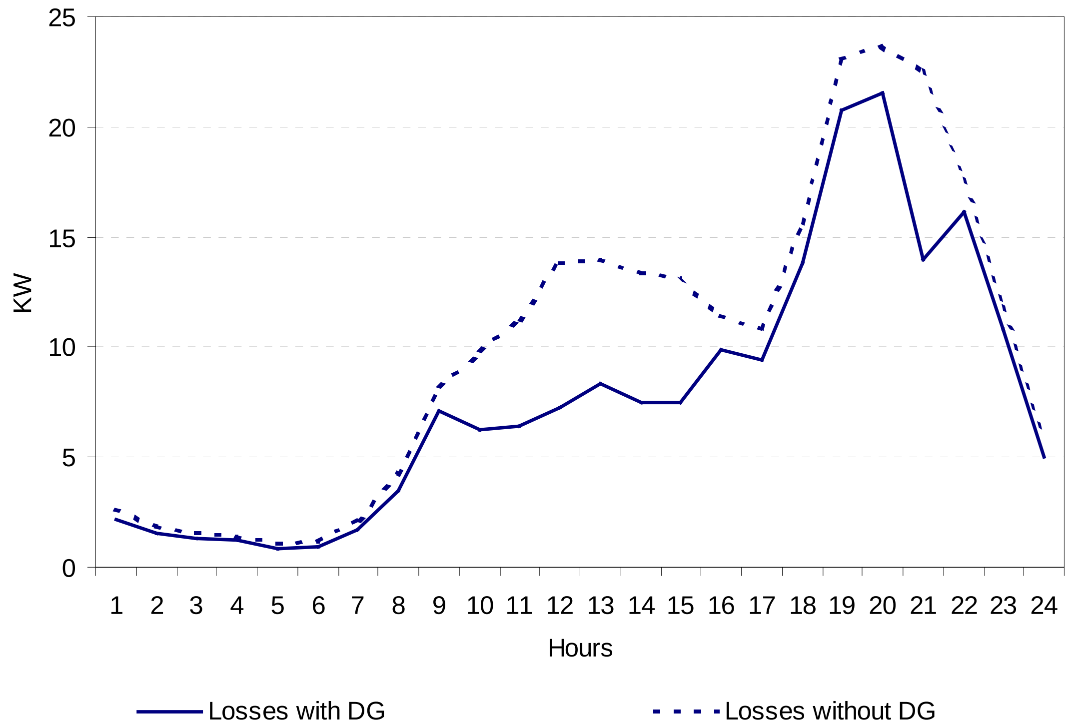


Fig. 3. Real power losses with and without DG power production

Applying next the described loss allocation method we evaluate the MLC's for the real power generation.

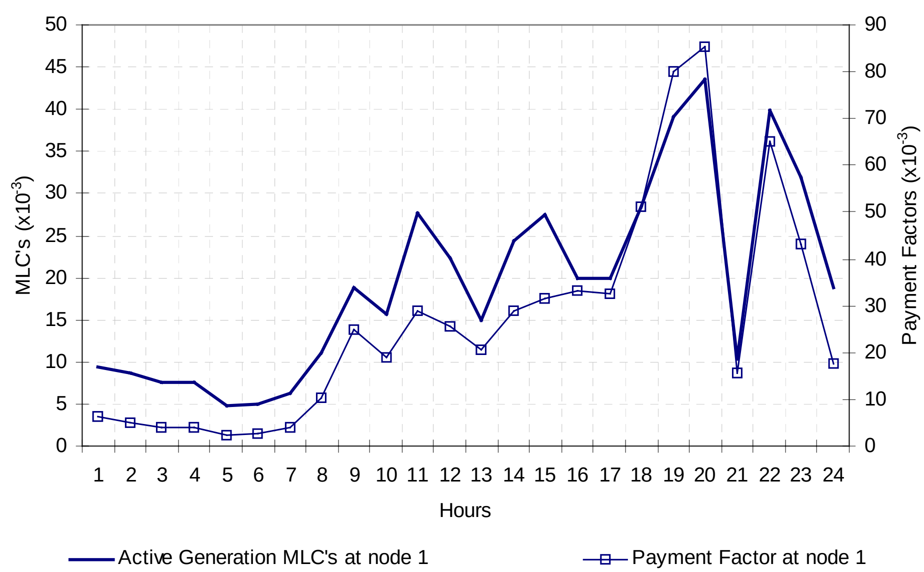


Fig. 4. MLC's related to active injections and payment factors at node 1 (slack bus)

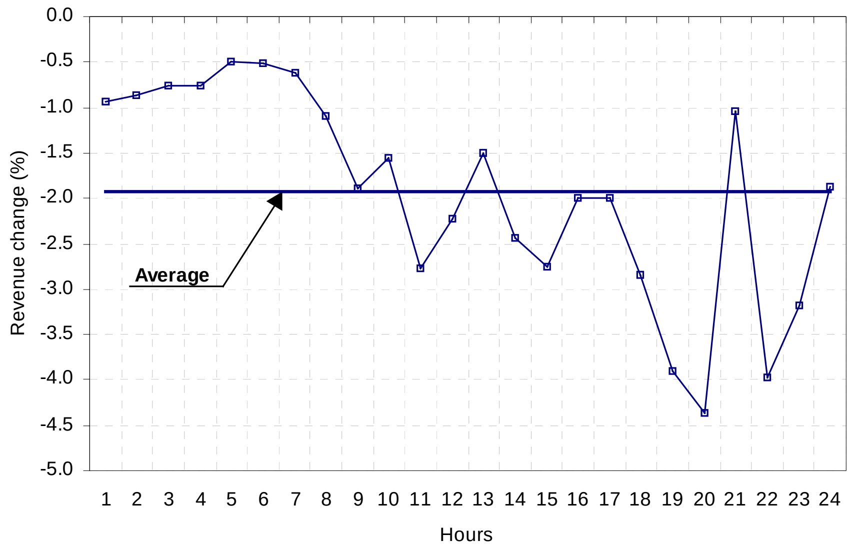


Fig. 5. Revenue change at node 1 (slack bus)

The real power injection from node 1 (slack bus) is always penalized, while the related MLC's are negative for the whole 24-time period and the average penalty is approximately 2%. This means that the revenue paid to the system for the real power injected to the LV network must be reduced by 2%.

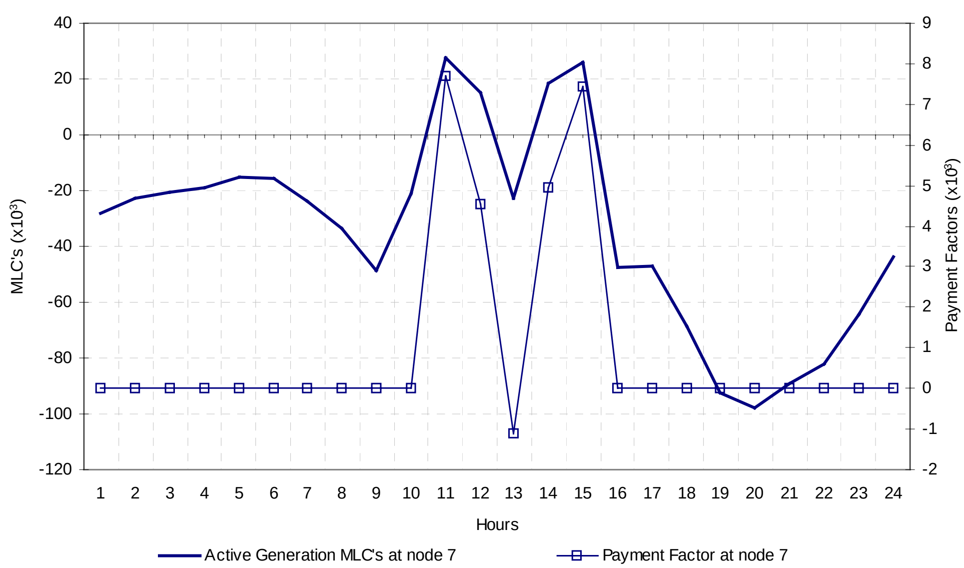


Fig. 6. MLC's related to active injections and payment factors at node 7

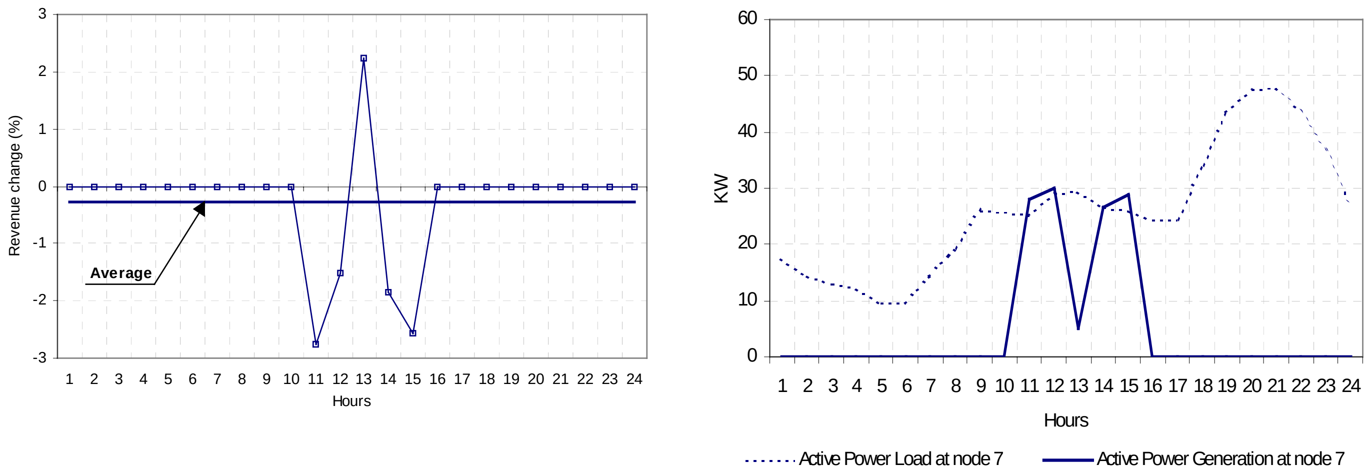


Fig. 7. Revenue changes at node 7 and real power generation and load profile at node 7

Node 7 is penalized in average by 0.27%. For the hours 11,12,14,15 the power producer (microturbine) is penalized as shown in fig. 7. (right part) due to real power generation over the load of the same node at the same time. At 13hour the producer is rewarded due to real power generation reduction following the load reduction (fig. 2) at industrial node (node 14) and at his own node 7.

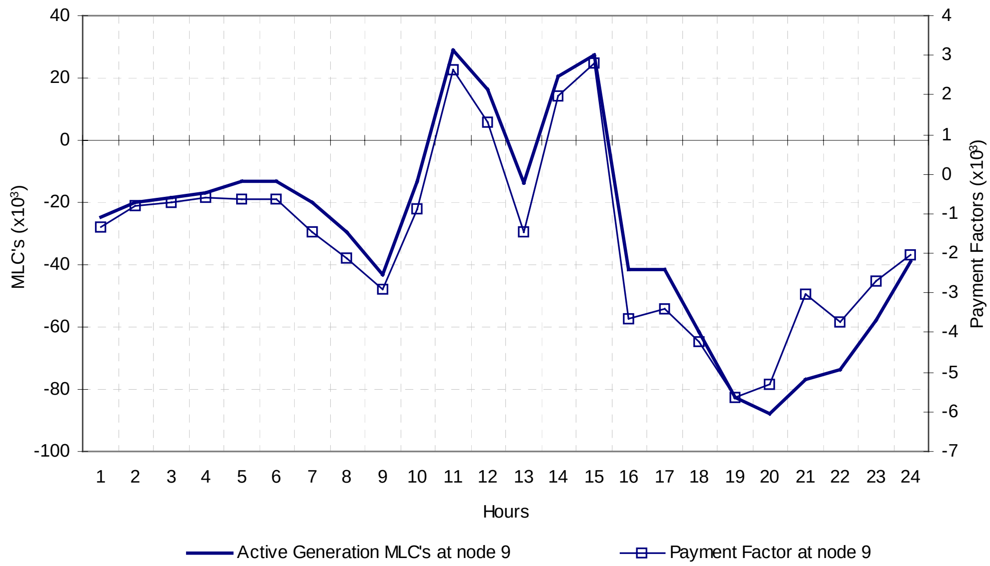


Fig. 8. MLC's related to active injections and payment factors at node 9

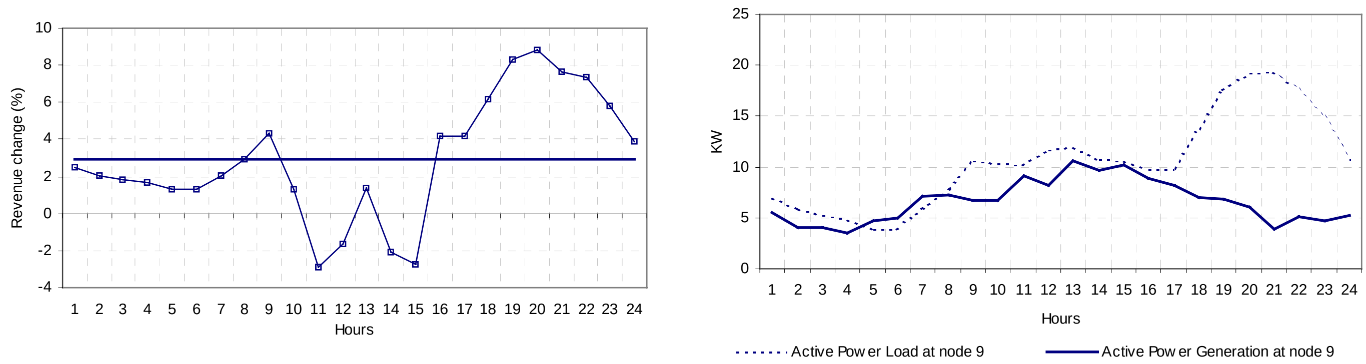


Fig. 9. Revenue changes at node 9 and real power generation and load profile at node 9

The producers at node 9 and 11 are penalized at the hours 11,12, 14, and 15 while they do not adjust their power production w.r.t. the production at node 7, thus injecting a significant amount of real power to the LV network. In contrary they are rewarded at the 13th hour, when they increase the power production while the producer at node 7 reduces his injection to the system.

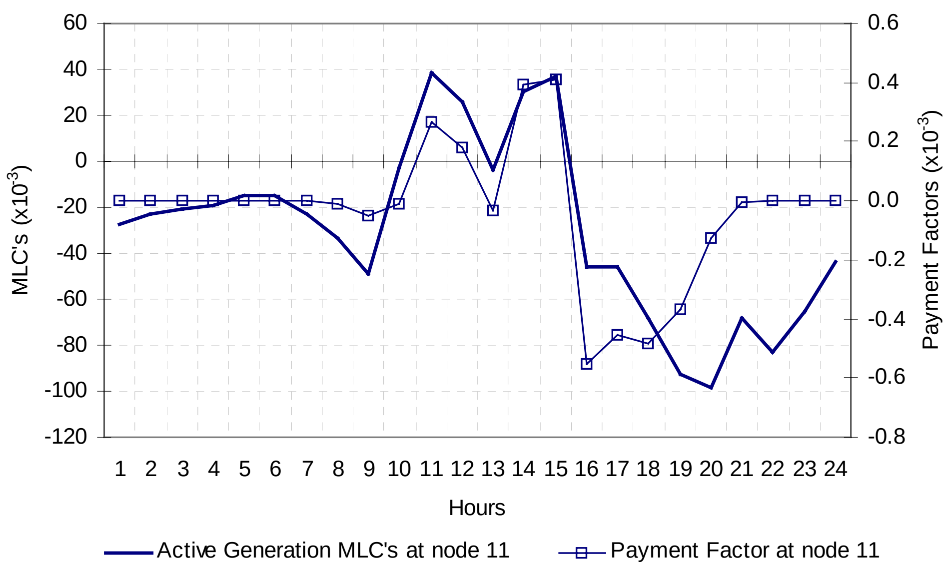


Fig. 10. MLC’s related to active injections and payment factors at node 11

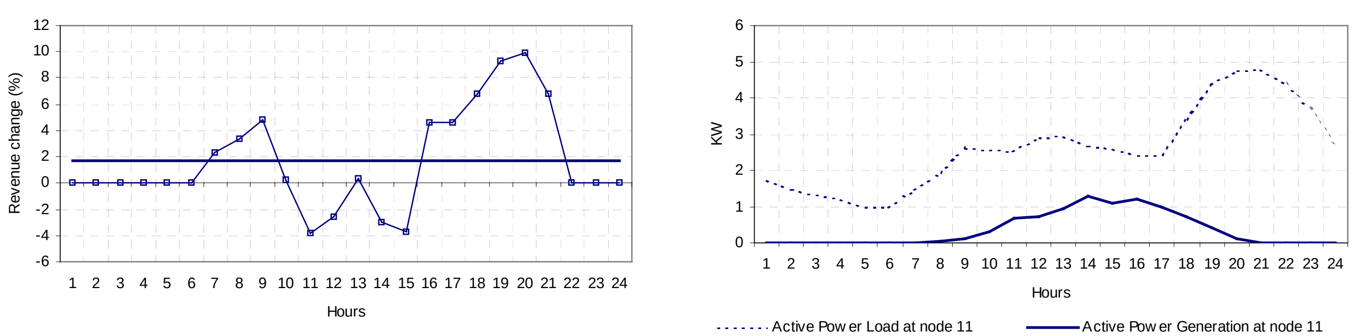


Fig. 11. Revenue changes at node 11 and real power generation and load profile at node 11

The power producer at node 13 is penalized at the hours 10-15 though for the hour 13 the penalty is quite lower than for the other hours. The generation for the hours 10-15 is stable and high and did not follow the reduction peak of the industrial load and generation at node 7. On the other hand, the producer is rewarded at the 21st hour, when he is injecting power to the system, while production at nodes 11 and 7 is switched off and the producer at node 9 reduces his generation at this time.

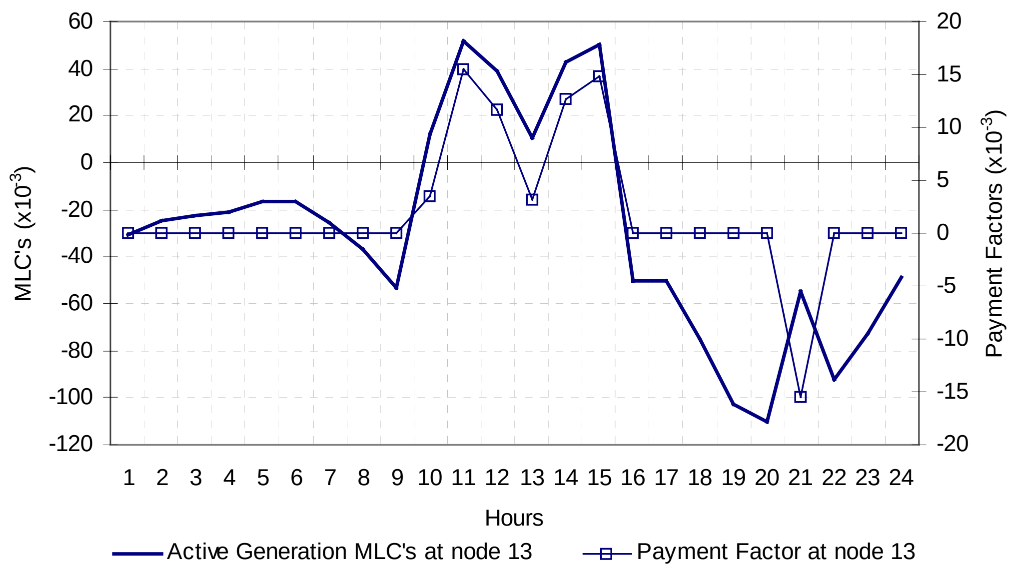


Fig. 12. MLC's related to active injections and payment factors at node 13

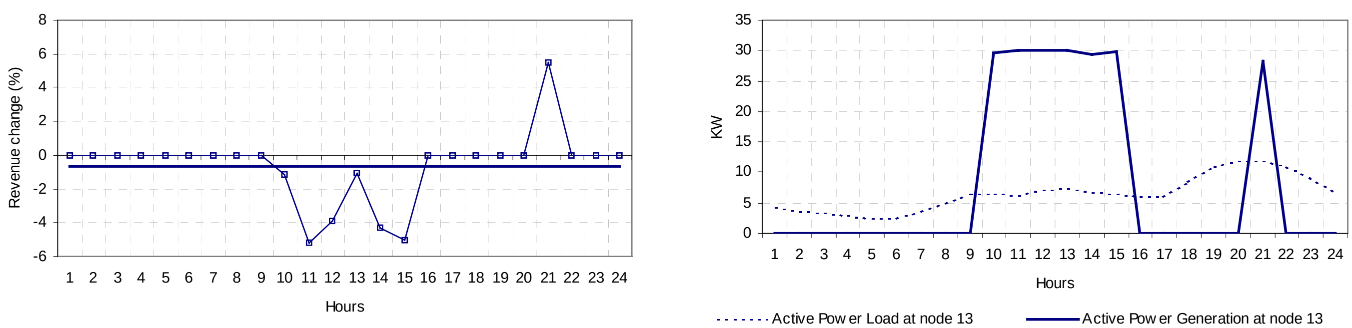


Fig. 13. Revenue changes at node 13 and real power generation and load profile at node 13

Figure 14 illustrates the change of the power consumption charge due to the allocation of losses. This means additional payments because of the losses they induce on the LV network. However, while the consumers are in average penalized, for some hours as shown in figure 15 they should be rewarded taking into account the reduction of losses. Thus the power consumption at the hours 11, 12, 14, and 15, when the producer at node 7 increases his production and the producer at node 13 maintains his power production stable, reduces losses as can be seen in figure 3.

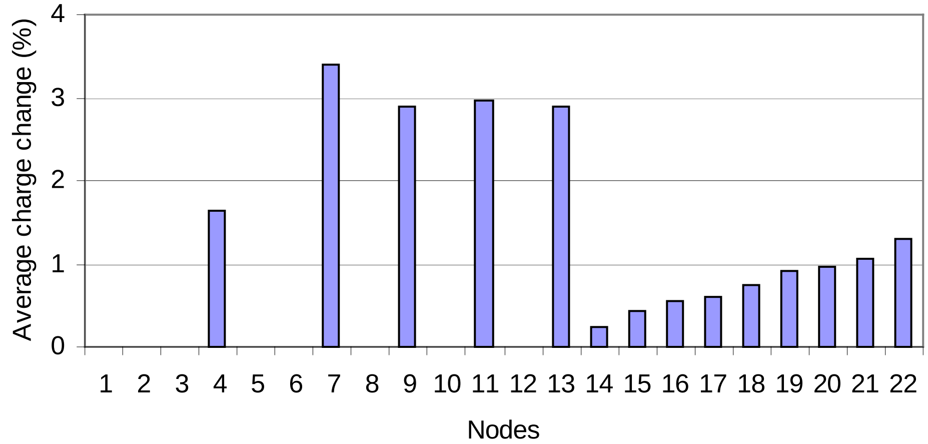


Fig. 14. Average charge changes at load nodes

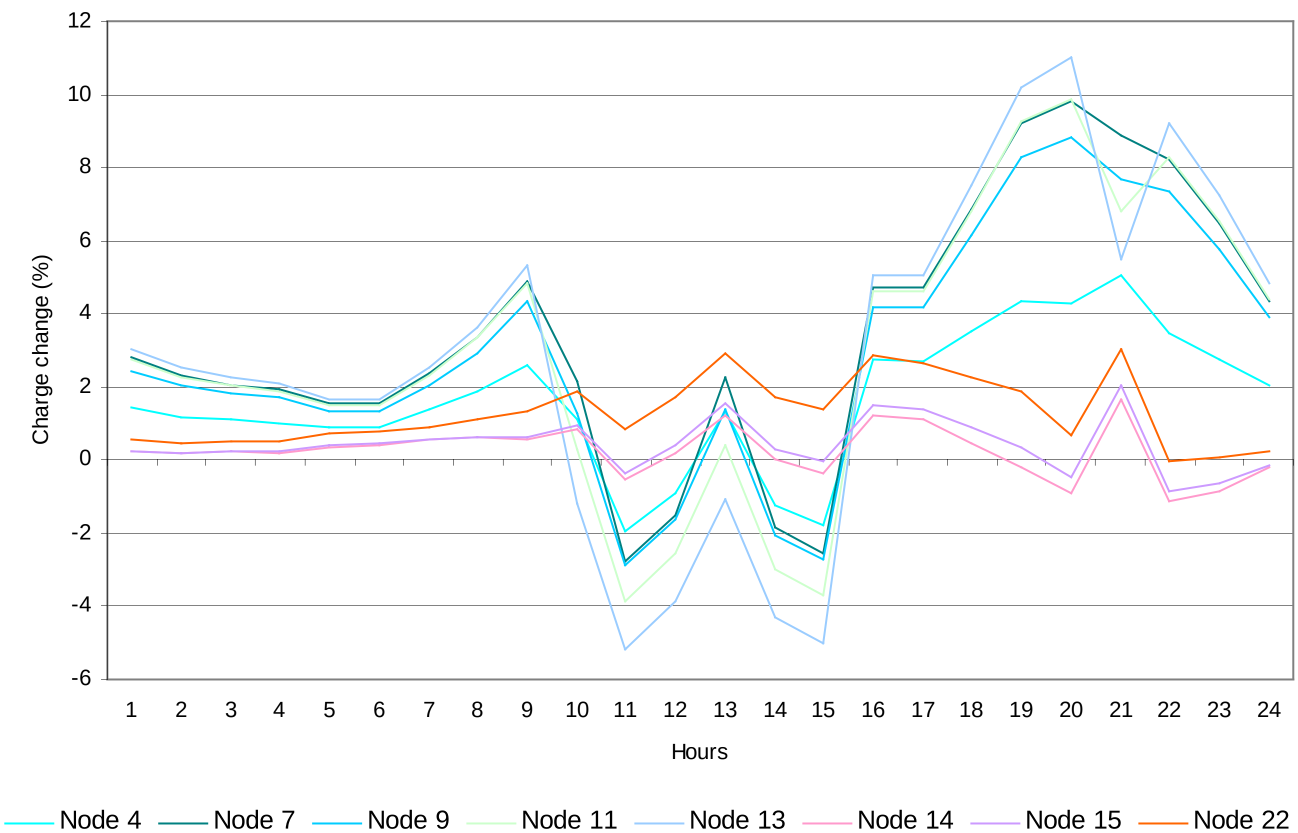


Fig. 15. Time variation of charge changes at load nodes

D. Conclusion

Loss allocation in LV networks with DG is a complex problem whose importance increases as competition in power generation encompasses smaller generation. This report presents a deterministic loss allocation method that is fair, transparent and provides appropriate signals to users of the network, neutralising the impact of choice of reference node on the magnitude and the polarity of loss allocation factors. The method is applied on a typical microgrid demonstrating its effectiveness.

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Large Scale Integration of Micro-Generation to Low Voltage Grids

Contract No. ENK5-CT-2002-00610

Work Package G

Report

Methods to Perform Loss Allocation Studies Review of the Literature and Simulations

Final version

May 2004

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Methods to Perform Loss Allocation Studies
Review of the Literature and Simulations

1. Introduction

1.1. Background

In the first steps of the re-regulation process implemented in power systems, losses were not an important topic since their value and their economic impact was reduced when compared with other issues. However, as the regulatory process develops, the allocation of losses gets a new importance both from the point of view of determining who will pay for them and eventually what agents have an adequate location so that they contribute to reduce losses. In this case, it is important to quantify this benefit so that these agents are prized for their location.

In the scope of the Microgrids projects it is important to address this issue in a robust way as well as adopting a method that is able to transmit economic signals to the grid users in order to induce more efficient uses of the grid and to induce the connection to more interesting locations. Apart from this, embedded generators can contribute to reduce losses and this can be interpreted as a service these generators are providing to the Microgrid. In this scope, a Microgrid in seen as an association of a LV distribution network, generators and loads together with controlling devices. In this sense, the Microgrid should recognise these agents that bring technical and economic advantages to the LV network and should prize them in an adequate way. This report addresses this subject and aims at contributing to solve this problem.

1.2. Structure of the report

After this introductory section, this report includes in section 2 a review of loss allocation methods described in the literature. This section starts with the enumeration of several principles that loss allocation methods should accomplish in terms of their technical robustness and their economic efficiency. The approaches described in section 2 include a description of expressions commonly used to approximate active losses, and methods as the proportional allocation, marginal allocation, proportional sharing allocation, allocation using the impedance matrix, incremental allocation, allocation based on the results of OPF studies and approaches developed to allocate active losses to transactions. Regarding the allocation based on the results of OPF studies, we describe two DC OPF models that are adapted in order to incorporate estimates of branch losses. Using the results of these optimisation problems it is possible to obtain marginal loss allocation coefficients. In this scope it is discussed an important topic related with the selection of the reference+slack bus and its impact on the values of those marginal loss allocation coefficients.

In section 3 we describe the method adopted to perform loss allocation studies specifically in distribution networks in the presence of embedded generators. This method is organised in three phases. The first one evaluates an operation point admitting that embedded generators are disconnected. The resulting are allocated to consumers. Secondly, it is run a second AC power flow considering that embedded generators are connected to the grid. This leads to the computation of the variations of active losses in all components of the grid to be allocated to embedded generators. In both cases, losses and the variations of losses are allocated to consumers and generators adopting the proportional sharing principle that states that the flows entering any node are distributed in a proportional way among the outflows. Finally, the third phase of this approach evaluates the loss variations due to voltage changes when the embedded generators are connected compared with the situation in which they are disconnected. These voltage related loss variations are allocated to generators adopting a simple pro-rata principle regarding the apparent power of these machines.

In section 4 we present the results obtained with the application of the approach described in section3 to the “Study-Case LV Network” whose characteristics are detailed in the document Study-Case LV-Network.pdf by Stavros Papathanassiou loaded in the Microgrids web page. Finally, section 5 enumerates the main conclusions obtained with this research.

2. Loss Allocation Methodologies Described in the Literature

2.1. General Issues

The structure of electricity markets adopted in several countries considers a centralised pool market as well as bilateral contracts. These markets are responsible for auctions for each hour of the next day. Generators and generations companies submit selling offers to the Market operator including hourly quantities and prices, while consumers and retailers submit buying offers including quantities and prices. On an hourly basis, the Market Operator builds aggregated buying and selling curves ordering buying offers in descending order of their prices and selling offers in ascending order of their prices. The intersection point of these two curves determines the market clearing price and the cleared quantity for each analysed hour.

This clearing process does not consider the impact of the grid and therefore losses are not considered in an explicit way in this process. However, during real time operation consumer measurement devices measure energy quantities actually absorbed and measurement devices installed in generation centers measure the energy that is generated, that is, consumer quantities plus losses in the grids. For this reason, it is important to know who will pay losses as well as defining the process to adopt to allocate losses to the entities that use networks or to the entities that, under a regulatory basis, are responsible for their payment. In a general way, both generators and consumers can be responsible for the payment of losses since both these entities use the grids and so both of them should be responsible for their payment. Losses are due to energy flows in the grids related to the dispatches obtained in a hourly basis in the pool, or related to the bilateral contracts.

Loss allocation to the entities that use the grids is not straightforward since losses are a non linear function of current magnitude in the branches and because physical laws that determine the operation of electricity circuits doesn't easily allow one to determine which are the energy flows determined by a particular generator or load. On the other side, several methods that use linearised expressions to allocate the flows to generators or loads face the difficulty in deciding the treatment to give to quadratic terms existing in exact expressions. This situation and these difficulties lead to the fact that there is not a definite and well-established way to allocate losses to generators and loads. In any case, any loss allocation approach should display a set of properties including:

- to be consistent with the results of power flow studies;
- depend on the produced energy, on the absorbed energy or on the injected currents in each node;
- depend on the relative location of each generator or load, regarding to the grid;
- lead to little volatile results;
- lead to results that transmit economic signals to the users of the grids;
- be easily understood;
- be easily implemented.

In this scope, the literature includes several papers and reports describing different approaches aiming at calculating losses and performing its allocation and tariffication. In the next points, we will review the available methodologies considering the following topics:

- expressions to use to evaluate losses in a grid depending on several electricity variables;
- methodologies to allocate losses to the generators or loads;
- methodologies to tariff the losses.

Apart from the loss allocation methods described in detail in the next sections, there are also tracing methods as the ones described in Bialek (1996-a), Bialek (1996-b) and Costa (2004). This class of methods is described in more detail in Section 3, since a method of this type was used in simulations to be presented in Section 4.

2.2. Expressions to Evaluate Active Losses in a Grid

Let us consider an electricity system including lines modelled by their equivalent π circuit, transformers represented by their reactance, loads represented by active and reactive powers and generators. In this way and considering the usual assumptions to perform an AC power flow study, the operation conditions of this grid are completely described by a set of variables that usually correspond to the voltage magnitudes and their respective phases. In these conditions, let us assume that a branch of this grid has:

- extreme nodes i and j;
- series resistance R_{ij};
- series reactance X_{ij};
- voltage magnitude in nodes i and j given by V_i and V_j;
- voltage phases in nodes i and j given by θ_i and θ_j;
- branch conductance given by (2.1).

$$G_{ij} = \frac{R_{ij}}{R_{ij}^2 + X_{ij}^2} \qquad (2.1)$$

Under these conditions, the active losses in branch i-j are given by (2.2).

$$Ploss_{ij} = G_{ij}.\left(V_i^2 + V_j^2 - 2.V_i.V_j.\cos(\theta_i - \theta_j)\right) \qquad (2.2)$$

This exact expression of the active losses in branch i-j can be subjected to several approximations that will be presented in the next points.

2.2.1. Approximation 1

Let us consider that they are valid the usual approximations inherent to the DC power Flow Model, namely that the voltage magnitudes are 1,0 pu. Apart from these approximations, let us also assume that the phase difference across branch i-j is sufficiently reduced so that the approximation of $\cos(\theta_i - \theta_j)$ given by (2.3) is valid.

$$\cos(\theta_i - \theta_j) = 1 - \frac{(\theta_i - \theta_j)^2}{2} \qquad (2.3)$$

Under these conditions, and admitting that the reactance of branch i-j is dominant regarding the resistance, we can obtain the approximate expression (2.4) by substituting (2.3) em (2.2), In this expression, $P_{ij,DC}$ corresponds to the active power flow in branch i-j computed according to the DC Power Flow Model, that is, by the division between the phase difference across branch i-j and the reactance of that branch.

$$Ploss_{ij} = \frac{R_{ij}}{X_{ij}^2}.(\theta_i - \theta_j)^2 = R_{ij}.\left(\frac{\theta_i - \theta_j}{X_{ij}}\right)^2 = R_{ij}.P^2_{ij,DC} \qquad (2.4)$$

2.2.2. Approximation 2

A second approximate expression for the active losses in branch i-j comes from using expression (2.2) admitting that the voltage magnitudes in nodes i and j is 1,0 pu. Under these conditions one obtains expression (2.5).

$$Ploss_{ij} = 2.G_{ij}.(1 - \cos(\theta_i - \theta_j)) \qquad (2.5)$$

2.2.3. Approximation 3

Another possibility that can be used to obtain an approximate expression to the active losses in branch i-j consists of running a DC dispatch study or a power flow study leading to the voltage phases in all network nodes. Using these values, it is possible to build a linearised expression that approximates the active losses in branch i-j considering the tangent line to the curve associated to expression (2.5) in the linearisation point corresponding to the operation point obtained when running the DC dispatch study or the power flow study already referred. In this case, for each branch i-j, one knows the following values:

- voltage phases in nodes i and j;
- approximate value of the active losses in branch i-j given by (2.6). This expression comes from (2.5) considering the operation point obtained by the DC dispatch study or the power flow study already referred. This operation point is denoted by p and corresponds to the referred linearisation point;

$$Ploss^p_{ij} = 2.G_{ij}.(1 - \cos(\theta^p_i - \theta^p_j)) \qquad (2.6)$$

- slope of the tangent line to the curve (2.5) in the point defined by θ^p_{ij} and $Ploss^p_{ij}$. This slope is given by (2.7) and it corresponds to the derivative of (2.5) computed in point p.

$$\frac{\partial Ploss_{ij}}{\partial \theta_{ij}}(\text{point } p) = 2.G_{ij}.\text{sen}(\theta^p_i - \theta^p_j) \qquad (2.7)$$

After computing the slope of this tangent line and using the linearisation point already referred, it is possible to use expression (2.8) corresponding the equation of the tangent line to

(2.5) in the referred point. The two coefficients in this expression are given by (2.9) and (2.10). In these expressions the index p designates the values computed at the end of the dispatch study or DC power flow study that was performed previously.

$$Ploss_{ij} \approx CL^1_{ij} + CL^2_{ij}.\theta_{ij} \tag{2.8}$$

$$CL^1_{ij} = 2.G_{ij}.(1-\cos\theta^p_{ij}) - (2.G_{ij}.\sin\theta^p_{ij}).\theta^p_{ij} \tag{2.9}$$

$$CL^2_{ij} = 2.G_{ij}.\sin\theta^p_{ij} \tag{2.10}$$

2.2.4. Approximation 4

One of the difficulties inherent to several dispatch models was related to fact that it was necessary to express the active losses in a grid in terms of the decision variables of the problem, that is, in terms of the generated powers. To overcome these difficulties it was developed an approximated expression known as B Coefficients, as it was detailed, for instance, in Saadat (1995). Briefly, the active losses in a grid can be expressed by (2.11).

$$Ploss = B_{00} + \begin{bmatrix} Pg_1 & Pg_2 & \cdots & Pg_{ng} \end{bmatrix} \begin{bmatrix} B_{01}/2 \\ B_{02}/2 \\ \vdots \\ B_{0ng}/2 \end{bmatrix} +$$

$$+ \begin{bmatrix} Pg_1 & Pg_2 & \cdots & Pg_{ng} \end{bmatrix} \begin{bmatrix} B_{11} & B_{12} & \dots & B_{1ng} \\ B_{21} & B_{22} & \dots & B_{2ng} \\ \vdots & \vdots & \ddots & \vdots \\ B_{ng1} & B_{ng2} & \dots & B_{ngng} \end{bmatrix} . \begin{bmatrix} Pg_1 \\ Pg_2 \\ \vdots \\ Pg_{ng} \end{bmatrix} \tag{2.11}$$

The computation of the B coefficients in this expression requires running an initial power flow study in order to compute voltage magnitudes and phases in all nodes. Using these values, it is possible to compute the currents that supply loads as well as the global load current. Afterwards, it must be built the Impedance Matrix as well as two other auxiliary matrices. Using all this information it is then possible to compute the B coefficients. One should notice that these coefficients depend on the operation point of the grid.. In any case, if there is a change of the generation dispatch to new values close to the original ones, the B coefficients will not change in a significative way.

2.3. Methodologies to Allocate Active Losses

2.3.1. Introduction

Let us consider a grid having generators and loads connected to it. The Energy Conservation Law indicates that the generated power should equal the load plus the active losses (2.12). In this expression, Pg and Pl are given by (2.13) and (2.14) in which Ng and Nl represent the number of generating nodes and demand nodes.

$$Pg = Pl + Ploss \qquad (2.12)$$

$$Pg = \sum_{i=1}^{Ng} Pg_i \qquad (2.13)$$

$$Pl = \sum_{i=1}^{Nl} Pl_i \qquad (2.14)$$

On the other side, in the next points we considered that there are one generator and one load at least in each node, so that we won't make any distinction between generator i, load i and node i.

2.3.2. Proportional Allocation

Proportional allocation corresponds to the simplest method to allocate active losses to generators and loads. In the first place, this method requires specifying the percentages of active losses to allocate to the set of generators and to the set of loads. Once these percentages are established, the allocation to each generator or to each load is performed in a proportional way regarding the total generated power, in the case of generators, or regarding the total load power, in the case of loads.

Assuming, for instance, that the set of generators and the set of loads are equally responsible for active losses, that is, the initial allocation percentages are 50%, one can obtain expressions (2.15) and (2.16) to allocate active losses to generator i and to load i. In these expressions $Plossg_i$ and $Plossl_i$ represent the active losses allocated to generator i and to load i.

$$Plossg_i = \frac{Ploss}{2} . \frac{Pg_i}{Pg} \qquad (2.15)$$

$$Plossl_i = \frac{Ploss}{2} . \frac{Pl_i}{Pl} \qquad (2.16)$$

This way, it is possible to compute the loss coefficients regarding generators and loads using (2.17) and (2.18). If these expressions are compared (2.15) and (2.16), respectively, Kg and Kl are given by (2.19) and (2.20).

$$Plossg_i = Kg.Pg_i \qquad (2.17)$$

$$Plossl_i = Kl.Pl_i \qquad (2.18)$$

$$Kg = \frac{1}{2} . \frac{Ploss}{Pg} \qquad (2.19)$$

$$Kl = \frac{1}{2} . \frac{Ploss}{Pl} \qquad (2.20)$$

One should stress that the loss allocation coefficients regarding generators, Kg, are equal for all grid nodes. In a similar way, the loss allocation coefficients regarding loads, Kl, are also equal for all nodes. On the other side, these coefficients are all positive. These aspects are simplifications of real power system operation and so one can conclude that these loss allocation coefficients do not reflect the operation conditions of power systems. It should be noticed that there are usually generators or loads in the system that have an adequate connection point from the point of view of contributing to reduce active losses. In a similar way, there are usually generators or loads in the system that have an bad connection point since they contribute to increase active losses. These aspects are not considered by this approach that, to a certain extent, could be denoted as Postage Stamp, in a similar way to the well known method to tariff the use of networks.

2.3.3. Marginal Allocation

Marginal loss allocation uses differential coefficients aiming at translating the impact in active losses due to a variation of the injected power, generated or load power, in a node i of the grid. Several publications on this topic, as for instance in Conejo et al (2002), these coefficients are called *Incremental Transmission Losses*, ITL, and are defined by (2.21).

$$K_i = \frac{\partial Ploss}{\partial \left(Pg_i - Pl_i\right)} \tag{2.21}$$

In this expression K_i corresponds to the *Incremental Transmission Loss* regarding node i. According to this definition, the ITL coefficient regarding the reference+slack node is zero. In fact, active losses in the grid will be compensated in that node, so that the variation of the injected power in the reference+slack node will have any impact in the branch power flows and therefore will not have any impact in active losses.

Using these coefficients, the active losses allocated to the generator connected to node i and to the load connected to node j are given by (2.22) and (2.23).

$$Plossg_i = Pg_i . \frac{\partial Ploss}{\partial Pg_i} = Pg_i . K_i \tag{2.22}$$

$$Plossl_j = Pl_j . \frac{\partial Ploss}{\partial Pl_j} = -Pl_j . K_j \tag{2.23}$$

However, due to the non linearities of the AC power flow equations, the addition of the losses allocated to the set of generators and to set of loads, P_{loss}^{marg}, coming from this type of allocation (2.24), is different from the active power that should in fact be allocated.

$$P_{loss}^{marg} = \sum_{i=1}^{Ng} Plossg_i + \sum_{j=1}^{Nl} Plossl_j = \sum_{i=1}^{Ng} Pg_i . K_i + \sum_{j=1}^{Nl} Pl_j . K_j \tag{2.24}$$

Having in mind this difference, the marginal loss coefficients regarding generators and loads must be normalised, so that they are able to allocate the correct amount of active power due to losses. This process is translated by (2.25). In this expression, the coefficients K_i^{marg} and K_j^{marg}, given by (2.26) and (2.27), are the normalised coefficients regarding nodes i and j.

$$Ploss = P_{loss}^{marg}.\frac{Ploss}{P_{loss}^{marg}} = \left(\sum_{i=1}^{Ng} Pg_i.K_i + \sum_{j=1}^{Nl} Pl_j.K_j \right).\frac{Ploss}{P_{loss}^{marg}} =$$

$$= \sum_{i=1}^{Ng} Pg_i.K_i^{marg} + \sum_{j=1}^{Nl} Pl_j.K_j^{marg} \tag{2.25}$$

$$K_i^{marg} = K_i.\frac{Ploss}{P_{loss}^{marg}} \tag{2.26}$$

$$K_j^{marg} = K_j.\frac{Ploss}{P_{loss}^{marg}} \tag{2.27}$$

Finally, the active losses allocated to each generator and load are given by (2.28) and (2.29).

$$P_{lossgi}^{marg} = Pg_i.K_i^{marg} \tag{2.28}$$

$$P_{perlj}^{marg} = -Pg_j.K_j^{marg} \tag{2.29}$$

According to these expressions, this methodology can produce positive or negative loss allocation coefficients. Some authors consider that negative variations can be interpreted as coming from a cross subsidy process between different entities.

This type of differential allocation requires the knowledge of an operation point of the system obtained, for instance, from a DC dispatch problem in which initially active are not taken into account. Using the results of this study, one can compute estimates of active losses in each branch of the system leading to the calculation of the differential coefficients K_i and K_j in (2.22) and (2.23). These coefficients should then be normalised using (2.26) and (2.27). Therefore, they depend on the operation point of the system already referred. They also depend on the node selected for reference+slack. This issue is relevant in this type of calculations given its immediate tariff impact. To overcome this problem Conejo et al (2002) refers a more complex and elaborated model that considers that the slack function is distributed among the generators in the system. This model is detailed in Galiana et al (2002).

2.3.4. Unsubsidised Marginal Allocation

The elimination of the cross subsidies detected in the previous approach can be obtained modifying the process used to compute the marginal loss coefficients, so that one avoids the computation of negative values. As a result, one obtains a set of marginal coefficients for generators and a set of marginal coefficients for loads. The reference Conejo et al (2002) explicitly refers that this method aims at allocating active losses in a grid and to explain physical facts, that is, there was not a concern in obtaining a procedure completely consistent with real world.

In the first place, the computation of marginal loss coefficients requires specifying a node for reference+slack, as referred previously. However, the coefficients computed for a different reference+slack node can be obtained from the previous ones using a translation coefficient defined in the interval $0{,}0 \le \beta \le 1{,}0$.

Let us start by considering that total active losses are given by (2.30). In this expression, N represents the number of nodes in the grid, K_i^{marg} represents the normalised marginal loss coefficient referred to node i, as computed by (2.26) or (2.27), and P_i is the injected power in node i, that is, the difference between generated and load power in that node.

$$Ploss = \sum_{i=1}^{N} K_i^{marg}.P_i = \sum_{i=1}^{N} K_i^{marg}.(Pg_i - Pl_i) \qquad (2.30)$$

On the other side, active losses can also be computed by (2.31), that is, by the difference between the sum of generations and the sum of loads.

$$Ploss = \sum_{i=1}^{N} P_i = \sum_{i=1}^{N} (Pg_i - Pl_i) \qquad (2.31)$$

If we multiply (2.30) by β, if we multiply (2.31) by $1-\beta$ and then add the two resulting expressions, we can see that total active losses Ploss can also be given by (2.32). Using (2.32) we can obtain (2.33). In this expression the term in the parenthesis can be interpreted as a new marginal loss coefficient given by (2.34).

$$Ploss = \sum_{i=1}^{N} \beta.K_i^{marg}.P_i + \sum_{i=1}^{N}.(1-\beta).P_i \qquad (2.32)$$

$$Ploss = \sum_{i=1}^{N} \left[\beta.K_i^{marg} + (1-\beta)\right].P_i = \sum_{i=1}^{N} K_i.P_i \qquad (2.33)$$

$$K_i = \beta.K_i^{marg} + (1-\beta) \qquad (2.34)$$

In what concerns generations, a change of the reference+slack bus should be performed so that the smaller marginal loss coefficients regarding generators comes to zero. To do this, let us consider that K_{Gk}^{marg} is the smaller normalised marginal loss coefficient regarding generators. In this situation, the translation coefficient β_G is computed using (2.35). If we solve (2.35) regarding β_G we obtain (2.36). Therefore, the new marginal loss coefficients regarding generators and referred to bus G (selected to reference+slack) are given by (2.37).

$$K_{Gk} = 0 = \beta_G.K_{Gk}^{marg} + (1-\beta_G) \qquad (2.35)$$

$$\beta_G = \frac{1}{1-K_{Gk}^{marg}} \qquad (2.36)$$

$$K_{Gi} = \beta_G.K_{Gi}^{marg} + (1-\beta_G) \qquad (2.37)$$

Regarding loads, the translation coefficient β_L is computed using (2.38) in which K_{Lm}^{marg} is the largest normalised marginal loss coefficient regarding loads. Solving (2.38) for β_L, the translation coefficient is given by (2.39) and the new marginal loss coefficients regarding loads and referred bus L are given by (2.40).

$$K_{Lm} = 0 = \beta_L . K_{Lm}^{marg} + (1 - \beta_L) \tag{2.38}$$

$$\beta_L = \frac{1}{1 - K_{Lm}^{marg}} \tag{2.39}$$

$$K_{Lj} = \beta_L . K_{Lj}^{marg} + (1 - \beta_L) \tag{2.40}$$

In a similar way to what was referred in the previous section, these coefficients depend on the operation point of the system, previously obtained through a dispatch problem. Afterwards, one should compute the marginal loss coefficients using the procedure detailed in the previous section. If any marginal loss coefficient regarding generators or loads is negative, one should perform a change of the reference+slack bus according to the previous indications.

2.3.5. Proportional Sharing Allocation

This approach to allocate active losses to generators and loads was developed by J. Bialek, based on the possibility of performing a tracing study of the electricity going from generators to loads. This approach is described in Bialek (1996-a), Bialek (1996-b), Reta et al (2001) and Conejo et al (2002). The main assumption of the proportional sharing allocation is that the flows entering any node are distributed proportionally between the outflows.

The loss allocation procedure is performed in a separate way for loads and for generators. Considering loads in the first place, it is defined the total load of the system as the sum of active demand powers and active losses (2.41). This global active power is also equal to the sum of the modified nodal loads, that is, the active powers connected to each node plus a term related with active losses (2.42).

$$P_L^{global} = P_L + Ploss \tag{2.41}$$

$$P_L^{global} = \sum_{j=1}^{NL} P_{Lj}^{Global} \tag{2.42}$$

On the other side, the global active power should be equal to the power supplied by generators, that is, the equilibrium equation (2.43) must hold. Adopting the proportional allocation principle, the power equilibrium in each node i of the grid can be expressed by (2.44) in which the coefficients c_{ji} are given by (2.45).

$$P_G = P_L^{Global} \tag{2.43}$$

$$P_i^{Global} = P_{Gi} + \sum_{j \in \alpha i} c_{ji} . P_j^{Global} \qquad \forall i = 1...N \tag{2.44}$$

$$c_{ji} = \frac{P_{ji}^{Global}}{P_j^{Global}} \approx \frac{P_{ji}}{P_j} \tag{2.45}$$

In these expressions:

- P_i^{Global} represents the global injected power in node i;
- P_{Gi} represents the generated power in node i;
- $\sum_{j \in \alpha i} c_{ji}.P_j^{Global}$ represents the power incident in node i considering all the lines node i is connected to;
- αi is the set of nodes from which depart lines connected to node i;
- P_{ji}^{Global} is the global power that flows from node j to node i;
- P_{ji} is the power that actually flows from node j to node i;
- P_j is the actually injected power in node j.

The set of equations (2.44) regarding each node i corresponds to a set of linear equations that can be solved for P_i^{Global}, for all the N nodes of the system. The global load and the active losses allocated to each node of the grid are computed using (2.46) and (2.47).

$$P_{Lj}^{Global} = \frac{P_j^{Global}}{P_j}.P_{Lj} \tag{2.46}$$

$$Ploss_{Lj} = P_{Lj}^{Global} - P_{Lj} \tag{2.47}$$

In a similar way, for the generators it is defined a global generated power using (2.48). This global power should also be equal to the sum of the generated powers in each node according to (2.49).

$$P_G^{global} = P_G + Ploss \tag{2.48}$$

$$P_G^{global} = \sum_{i=1}^{NG} P_{Gi}^{Global} \tag{2.49}$$

This global power should equal the global load power, that is, the equilibrium equation (2.50) must hold. Using again the referred proportional allocation principle in defining the power equilibrium for node i, we can obtain (2.51). This equation for node I means that the injected power in node i is equal to the load connected to that node plus the sum of powers that flow away from it in all lines it is connected to.

$$P_L = P_G^{Global} \tag{2.50}$$

$$P_i^{Global} = P_{Li} + \sum_{j \in \gamma i} c_{ji}.P_j^{Global} \qquad \forall i = 1...N \tag{2.51}$$

In this expression:

- P_i^{Global} represents the global injected power in node i;
- P_{Li} represents the load power in node i;
- $\sum_{j\in\gamma i} c_{ji}.P_j^{Global}$ represents the sum of the powers that flow away of node i considering all the lines it is connected to;
- γi is the set of nodes from which depart lines connected to node i.

The set of equations (2.51) regarding each node i forms a set of linear equations that can be solved for P_i^{Global} for all the N nodes of the grid. Using these values, we can compute the new generation values and the active losses using (2.52) and (2.53).

$$P_{Gi}^{Global} = \frac{P_i^{Global}}{P_i}.P_{Gi} \tag{2.52}$$

$$Ploss_{Gi} = P_{Gi} - P_{Gi}^{Global} \tag{2.53}$$

Considering a regulatory point of view, we can also specify the percentage of active losses to allocate to the generators and to the loads. For instance, if we want to allocate 50% of the active losses to the generators and the remaining 50% to loads, the nodal generator and load values should be computed by (2.54) and (2.55). Then, the final value of active losses allocated to each generator and to each load is given by (2.56) or by (2.57).

$$P_{Gi}^{new} = \frac{P_{Gi}^{Global} + P_{Gi}}{2} \tag{2.54}$$

$$P_{Lj}^{new} = \frac{P_{Lj}^{Global} + P_{Lj}}{2} \tag{2.55}$$

$$Ploss_{Gi}^{new} = P_{Gi} - P_{Gi}^{new} \tag{2.56}$$

$$Ploss_{Lj}^{new} = P_{Lj}^{new} - P_{Lj} \tag{2.57}$$

The active losses allocated to each generator or load given by (2.56) or (2.57), can also be used to obtain loss allocation coefficients K_{Gi} and K_{Lj}, using (2.58) and (2.59).

$$Ploss_{Gi}^{new} = P_{Gi} - P_{Gi}^{new} = K_{Gi}.P_{Gi} \tag{2.58}$$

$$Ploss_{Lj}^{new} = P_{Lj}^{new} - P_{Lj} = K_{Lj}.P_{Lj} \tag{2.59}$$

Solving each of these two expressions for the coefficients K_{Gi} and K_{Lj}, we obtain the loss allocation coefficients to generators and loads given by (2.60) and (2.61).

$$K_{Gi} = 1 - \frac{P_{Gi}^{new}}{P_{Gi}} \tag{2.60}$$

$$K_{Lj} = \frac{P_{Lj}^{new}}{P_{Lj}} - 1 \tag{2.61}$$

The application of this allocation approach is based on a tracing principle of electricity from generators to loads. In meshed grids performing such a study requires additional information since it is not possible to establish in an unique way the paths of the energy from generators to loads. In this approach, the additional information corresponds in this case to the adoption of the proportional allocation principle. Under these conditions, once an operation point of the system is obtained (using a power flow study or a dispatch formulation), and once the referred proportional allocation principle is accepted, one should specify the allocation percentages of losses to the set of generators and to the set of loads. Finally, one computes the allocation coefficients by (2.60) and (2.61).

2.3.6. Allocation Using the Impedance Matrix

Several references proposed using the nodal impedance matrix to perform a loss allocation study for the nodes of a grid. (Conejo et al (2001), Moyano et al (2002) and Lima et al (2002)). According to this approach, one aims at using the results of a power flow study in order to allocate in a systematic way the active losses by the N nodes of the system, considering that condition (2.62) must hold.

$$\text{Ploss} = \sum_{k=1}^{N} \text{Ploss}_k \tag{2.62}$$

In this expression, the terms Ploss_k correspond to the part of total active losses that will be allocated to the injected power in node k. These values can then be used to allocate to entities connected to each node k the responsibility for the payment of Ploss_k valued at the system marginal price. If a node k has a generator and a load connected to it, the cost of losses allocated to that node can then be split by these two entities proportionally to the respective powers.

To compute the terms Ploss_k let us consider in the first place that the nodal admittance matrix Y=G+jB is sparse and non-singular. Let us also consider that we have the complete AC results of a power flow study, namely the vector of nodal injected currents, I, and the vector of nodal voltages. The losses in the grid can then be expressed in terms of the Y matrix and vector V, or in terms of the Z matrix and vector I. In these conditions, active losses can be given by (2.63) from which we obtain (2.64) or (2.65).

$$\text{Ploss} = \text{Real}\left\{ \sum_{k=1}^{N} V_k . I_k^* \right\} \tag{2.63}$$

$$\text{Ploss} = \text{Real}\left\{ \sum_{k=1}^{N} V_k . \left(\sum_{j=1}^{N} Y_{kj}^* . V_j^* \right) \right\} \tag{2.64}$$

$$Ploss = Real\left\{\sum_{k=1}^{N} I_k^{*}.\left(\sum_{j=1}^{N} Z_{kj}.I_j\right)\right\} \tag{2.65}$$

The formulation adopting (2.65) is used more frequently since it directly depends on the injected currents in each node, so that one of the requirements stated in section 2.1 for a loss allocation approach. The allocation process based on (2.65) admits that it is possible to separate this expression in two sums. One of them is related with the resistance matrix, R, and the other one is related with the reactance matrix, X. Under these conditions, expression (2.65) leads to (2.66). In reference Conejo et al (2001) it is demonstrated that the second term is zero so that the active losses can be only expressed in terms of the resistance matrix and the complex injected currents.

$$\begin{aligned} Ploss &= Real\left\{\sum_{k=1}^{N} I_k^{*}.\left(\sum_{j=1}^{N} R_{kj}.I_j\right)\right\} + Real\left\{\sum_{k=1}^{N} I_k^{*}.\left(\sum_{j=1}^{N} jX_{kj}.I_j\right)\right\} = \\ &= Real\left\{\sum_{k=1}^{N} I_k^{*}.\left(\sum_{j=1}^{N} R_{kj}.I_j\right)\right\} \end{aligned} \tag{2.66}$$

According to this expression, it is immediate to allocate active losses to each node k using (2.67). According to (2.67), the term of active losses to allocate to node k, $Ploss_k$, includes N terms representing the couplings between injected currents in each of the N nodes with the injected current in node k.

$$Ploss_k = Real\left\{I_k^{*}.\left(\sum_{j=1}^{N} R_{kj}.I_j\right)\right\} \tag{2.67}$$

This loss allocation approach only requires knowing an operation point of the system fully characterised, for instance from an AC power flow study. Once this point is computed, it is not necessary to consider any other kind of assumption or approximation since this approach uses the AC exact power flow equations that express the operation of the system considering the vector of nodal voltages and injected currents together with the impedance matrix.

2.3.7. Incremental Allocation of Active Losses

Several references describe loss allocation approaches based on the sequential solution of AC power flow studies (Galiana et al (2000), Moyano et al (2002) and Galiana et al (2002). This kind of approaches admit that an infinitesimal variation of a bilateral transaction or of the dispatch obtained in the pool market also leads to an infinitesimal variation of the injected powers on buses involved in that transaction.

Given that this kind of approaches is based on the solution of several power flow studies, the variation corresponding to a set of infinitesimal variations of the injected power ∂P_j, will be completely reflected in the power of the bus selected for reference+slack. This change can be computed using (2.68).

$$\partial Ploss = \sum_{j=1}^{N} \frac{\partial Ploss}{\partial P_j} . \partial P_j \tag{2.68}$$

In this expression the derivative of the active losses regarding the active power in node j corresponds to the *Incremental Transmission Loss*, ITL, as it was referred in section 2.3.3. These coefficients assume non-zero values, except for the bus selected for reference+slack. In fact, if there is an infinitesimal change of the injected power in the reference+slack node there isn't any change in the branch power flows and current magnitudes. Therefore, the derivative of that injected power regarding losses, Ploss, is zero.

For an increment of the power related to a bilateral contract established between a generator connected to node r and a load connected to node s, the corresponding change in the active losses is given by (2.69).

$$\partial Ploss = \left(\frac{\partial Ploss}{\partial P_r} - \frac{\partial Ploss}{\partial P_s} \right) \partial GL_{rs} \tag{2.69}$$

In this expression, ∂GL_{rs} represents the infinitesimal change in the contracted power between nodes r and s. This infinitesimal change is multiplied by the derivative of active losses regarding the generated power in node r and by the symmetric of the derivative regarding the load in node s. The positive sign affecting the first derivative and the negative one affecting the second one comes from the fact that expression (2.68) was originally established in terms of the injected power.

The procedure to adopt to allocate active losses to each contract consists of incrementing each bilateral contract in a gradual way, for instance by steps of magnitude ∂GL_{rs}. Afterwards, it will be used expression (2.69) to evaluate the impact of those variations in active losses. Finally, expression (2.70) will be used to update generated powers.

$$Pg_i = \sum_{j=1}^{N} GL_{ij} + \sum_{r,s=1}^{N} Ploss_{rs,\,j} \tag{2.70}$$

The power associated to each generator is divided in steps that will be successively added till one reaches the total contracted power by all generators. After performing such an addition, it must be solved a new power flow study in order to refresh the voltage magnitudes and phases, the active losses and the derivatives of active losses regarding injected powers. This procedure is implemented till reaching the desired generation and load level.

This approach has a number of drawbacks such as:

- it is necessary to specify a bus for slack. The results of the loss allocation will depend on this selection;
- if the number of bilateral contracts is large or there is a large number of generated and load powers dispatched in the pool, it will be necessary to run a time consuming procedure that includes an initial power flow study and a new power flow study whenever there is a change in generated or load powers;
- as it is easily understood, the adopted step ∂GL_{rs} can be enlarged to shorten this time consuming procedure. This strategy will certainly lead to poor results since the

derivative procedure corresponds to compute a linearisation point of the active loss expression that will be affected by an enlarged variation of the injected power;

- finally, it is important to refer that if the number of bilateral contracts is large it should also be decided the order to adopt to consider the changes ∂GL_{rs} of the generated and load powers inherent to each of them. If one considers different orders, the results obtained with this approach will also be different since the operation points computed by the successive power flow studies will not be the same. This also reflects the non linear nature of the AC power flow equations. From a regulatory point of view this issue is highly undesirable since the results will depend on a pre-specified order opening way to subjectivity and lack of transparency.

2.3.8. Loss Allocation Based on the Results of OPF Problems

The use of Optimal Power Flow, OPF, based formulations to perform active losses allocation studies is also described in several references (Rivier et al (1990), Rivier et al (1993), Rivier et al (1994) and Finney et al (1996)). Some of these approaches are based on the AC power flow model while some others adopt the DC model.

Regarding the approaches that use the DC model, they are used several expressions to obtain estimates of active losses that, when included in these formulations, usually lead to iterative procedures. These approaches usually adopt the concept of nodal marginal price of electricity reflecting the impact in the cost function of the optimisation problem from changing the load in one node of the grid.

If one adopts a non linear formulation, we can then use a Lagrangean approach in which the objective function is modified in order to incorporate information about the constraints. This objective function can then be optimised using a simple gradient based technique or some other more elaborated optimisation method. In any case and as a subproduct of the solution of this problem, we can obtain the Lagrange multipliers that can be used to compute the nodal marginal prices of electricity.

Depending on the adopted formulation, these marginal prices can be decomposed in the marginal price on the node selected for reference+slack, on a component reflecting the marginal loss variations and on a component related with congestion in branches of the grid.

The linearised approaches based on the DC model and in the integration of estimates for active losses are usually considered as a good compromise between more elaborated formulations using the AC power flow equations and single node or multi node formulations that do not consider active losses. In these formulations the nodal marginal prices are obtained adequately combining the dual variables of the problem computed when the optimal solution is identified. The importance of this type of approaches and its use in several tariff systems justify that we give them a particular attention in section 2.4.

Regarding to these loss allocation approaches, it is important to refer:

- in general, the loss coefficients depend on the bus selected for slack. However, as this bus usually coincides with the reference bus, it is usual to consider that marginal loss coefficients depend on the bus selected for reference+slack;
- usually, a good loss allocation, or in other words, a loss allocation closer to the actual operation conditions of the system, can be obtained if the reference+slack bus is

adequately selected. In fact, due to the adopted mathematical formulations, the marginal loss variation is compensated in the slack bus that, as it was referred, usually coincides with the reference bus. In real systems, loss variations are compensated in the bus where it is connected the marginal generator. This means that if the bus where it is connected the marginal generator is selected for reference+slack, the previous problem does not exist anymore;.

- in fact, it is not always easy or even possible to select the bus for reference+slack as the bus where it is connected the marginal generator. In the first place, the marginal generator is usually very volatile. This means that the grid can impose congestion situations so that the marginal generator is one for load variations in some buses and is another one for load variations in some other buses. Secondly, this generator can also change if there are outages (either branch or generator outages) or if the topology in operation changes. Thirdly, the marginal generator function can in fact be distributed by several machines. This means that part of the marginal load or loss variation is compensated in one generator and another part is compensated in another one. Under these conditions, it is not possible to get a coincidence between the reference+slack bus and the bus where it is connected the marginal generator;
- besides performing a marginal loss allocation, these methods can also be used to tariff those losses. In fact, when computing nodal marginal prices we are implicitly indicating that loads should the losses they are responsible for and that generators should receive a remuneration related with the marginal price in the node they are connected to. Therefore, this type of approaches has an important advantage since it inherently sends economic signals to the users of the grid so that this grid is used in a more efficient way.

Therefore and to conclude this section, it should be referred that if it is possible to overcome the difficulty of selection of the reference+slack bus, the marginal based methods are the most robust ones from a technical point of view, they reflect the operation conditions of the systems and they are able to transmit economic signals to the users of the grids.

2.3.9. Allocation of Losses to Transactions

Several recent papers addressed the problem of allocating active losses to a set of transactions. In this scope, Expósito et al (2000), Tao et al (2000) and Fradi et al (2001) described several approaches to perform this allocation.

An example, Expósito et al (2000) considers that in an electric grid in which there are t transactions, branch active losses can be approximately given by (2.71).

$$\text{Ploss} \cong \left(\sum_{i=1}^{t} P_i \right)^2 .R = \sum_{i=1}^{t} P_i^2 .R + \sum_{\substack{i,j=1 \\ i \neq j}}^{t} 2.P_i .P_j .R \tag{2.71}$$

In this expression, P_i represents the active power flow in the branch under analysis due o transaction i and R represents the resistance of that branch. This expression indicates that the active losses in this branch has two components. The first one, related to the first sum in

(2.71) does not present any difficulty in performing its allocation since each term of this sum will be allocated to each transaction i.

The second term in this expression places several difficulties regarding its allocation to transaction i. In fact, this sum represents a set of cross terms, that is, depends on the product of the power flows in the branch under analysis due to pairs of transactions.

The paper Expósito et al (2000) presents several ways to perform the separation of these cross terms. In the place, if we only consider transactions i and j and if we want to separate the term $P_i.P_j$ it should hold an equilibrium relation between the two resulting terms and the global power to separate. This equilibrium relation can be modelled by (2.72). In this relation β_i and β_j represent the allocation coefficients of the term $2.P_i.P_j$ to the transaction i and j. This relation can be finally translated by (2.73).

$$\beta_i (P_i.P_j) + \beta_j (P_i.P_j) = 2.P_i.P_j \tag{2.72}$$

$$\beta_i + \beta_j = 2 \tag{2.73}$$

This relation does not indicate an unique way to perform this allocation, since we have two variables, the allocation coefficients, and we only have a single mathematical relation between them. This paper presents several alternative ways to perform this allocation taking into account (2.73) and imposing additional constraints to determine the allocation coefficients β_i and β_j.

2.3.9.1 Proportional Allocation

The proportional allocation is the simplest way to perform loss allocations to transactions. This approach is based in expression (2.74).

$$\frac{\beta_i}{P_i} = \frac{\beta_j}{P_j} \tag{2.74}$$

According to the previous description, P_i and P_j represent the active power flows in the branch under analysis due to the transactions i and j, and β_i and β_j represent the loss allocation coefficients to the transactions i and j. The relations (2.73) and (2.74) correspond to a set of equations that can be solved for the allocation coefficients β_i and β_j. The solution of the system leads to (2.75) and (2.76) for these two allocation coefficients. In these expressions, P_a represents the average between the powers P_i and P_j.

$$\beta_i = \frac{P_i}{P_a} \tag{2.75}$$

$$\beta_j = \frac{P_j}{P_a} \tag{2.76}$$

2.3.9.2 Quadratic Allocation

The quadratic allocation of losses to transactions i and j is based on the expression (2.77). The adoption of this additional constraint comes from the fact that active losses depend on the active power flows in a quadratic way.

$$\frac{\beta_i}{P_i^2} = \frac{\beta_j}{P_j^2} \tag{2.77}$$

The combination of this expression with (2.73) leads to a new set of linear equations that can be solved for the allocation coefficients leading to (2.78) and (2.79). These allocation coefficients are related to the two transactions, i and j, that we are considering.

$$\beta_i = \frac{P_i^2}{\frac{1}{2}.\left(P_i^2 + P_j^2\right)} \tag{2.78}$$

$$\beta_j = \frac{P_j^2}{\frac{1}{2}.\left(P_i^2 + P_j^2\right)} \tag{2.79}$$

2.3.9.3 Geometric Allocation

As the term whose decomposition we are analysing corresponds to the product of two factors, the geometric decomposition uses an intermediate variable P_g given by (2.80) as well as the logarithmic relation (2.81).

$$P_g = \sqrt{P_i . P_j} \tag{2.80}$$

$$\beta_i - \log P_i = \beta_j - \log P_j \tag{2.81}$$

Using these two relations, we can obtain expressions (2.82) and (2.83) for the loss allocation coefficients.

$$\beta_i = 1 + \frac{1}{2}.\log\left(\frac{P_i}{P_j}\right) = 1 + \log\left(\frac{P_i}{Pg}\right) \tag{2.82}$$

$$\beta_j = 1 + \frac{1}{2}.\log\left(\frac{P_j}{P_i}\right) = 1 + \log\left(\frac{P_j}{Pg}\right) \tag{2.83}$$

These coefficients should be only used when conditions (2.84) hold since these coefficients must be according to (2.73). This means that individually they must be in the interval $[0{,}0;2{,}0]$.

$$0{,}01 \le \frac{P_i}{P_j} \le 100{,}0 \tag{2.84}$$

The relations (2.82), (2.83) and (2.84) can be generalised in order to accommodate more transactions, considering the quotients between P_i and P_j. Under these conditions, one can use expressions (2.85) and (2.86) if condition (2.87) holds for any pair of transactions. In expressions (2.85) and (2.86), r represents the number of transactions under consideration, and P_i and P_j represents the branch power flows in the branch under analysis due to transactions i and j, respectively.

$$\beta_i = 1 + \frac{1}{r}.\log\left(\frac{P_i}{P_j}\right) \tag{2.85}$$

$$\beta_j = 1 + \frac{1}{r}.\log\left(\frac{P_j}{P_i}\right) \tag{2.86}$$

$$10^{-r} \le \frac{P_i}{P_j} \le 10^r \tag{2.87}$$

2.3.9.4 Fast Geometric Allocation

If the number of transactions is large, the previous loss allocation techniques based on the separation of cross terms gets too much time consuming. In this case the Geometric Allocation Technique can be modifies so that the active losses in a branch can be allocated in a more efficient way. In this scope, let us assume that P_M represents the largest magnitude of the power flow sue to any of the transactions in analysis (2.88). Let us also consider that fall power flows for which condition (2.89) are not considered, that is, no losses are allocated to these transactions. Under these conditions, relation (2.84) can be used for any pair of the remaining transactions.

$$P_M = \max\left|P_i\right| \tag{2.88}$$

$$\left|P_i\right| \le 10^{-r}.P_M \tag{2.89}$$

The active losses allocated to transaction i can then be given by (2.90) or by (2.91) after rearranging the terms in (2.90). In (2.91), the power P_l is given by (2.92). Expression (2.90) comes from the generic expression (2.71) in which we considered the relation (2.85) and in which we didn't consider the terms for which (2.89) hold.

$$Ploss_i = \sum_{j=1}^{t}\left[1 + \frac{1}{r}.\log\left(\frac{P_i}{P_j}\right)\right].P_i.P_j.R \tag{2.90}$$

$$Ploss_i = P_i.\left(\sum_{j=1}^{t} P_j\right).\left(1 + \frac{1}{r}.\log P_i\right)R - \frac{1}{r}.P_i.P_l.R \tag{2.91}$$

$$P_l = \sum_{j=1}^{t} P_j.\log P_j \tag{2.92}$$

On the other hand, Fradi et al (2001) describe a methodology to allocate active losses to transactions that is based on the computation of allocation coefficients of branch active losses to transactions. According to this reference, the active losses are given by (2.93) in which $\eta_{ramoi-j,t}$ represents the loss coefficient of branch i-j losses allocated to transaction t and P_t is the power involved in that transaction. These coefficients must be set according to (2.94).

$$Ploss_{ij} = \sum_t \eta_{branchi-j,t}.P_t \tag{2.93}$$

$$\sum_t \eta_{branchi-j,t} = 1 \tag{2.94}$$

Once the active losses for line i-j are allocated to each transaction, it is possible to perform the allocation in a wider area, by simply adding the losses in each line in that area (2.95).

$$Ploss_{area\ n} = \sum_{linhas\ área\ n} Ploss_{i-j} = \sum_{linhas\ área\ n} \sum_t \eta_{ramoi-j,t}.P_t \tag{2.95}$$

In this formulation, one transaction corresponds to the generated power P_t in node p_t and it absorption in node q_t. Let us also consider that there are T transactions to implement by using the electricity grid. Let us admit that we performed a power flow study for a base situation corresponding to the absence of the T transactions. This power flow study provides the voltage magnitudes and phases in all nodes, In this base situation we admit it is possible to know the active losses in terms of voltage magnitudes and phases, P^o_{loss}.

Afterwards, we will add the powers included in the referred T transactions. When all transactions were added, the initial active losses, P^o_{loss}, change to $P^o_{loss} + \Delta Ploss$. In this case we want to allocate the term $\Delta Ploss$ to each of the T transactions.

To perform this computation, this reference admits that each transaction P_t is proportional to a parameter s that varies in the interval $[0,0;1,0]$, so that a transaction can be represented by (2.96). If the derivative of $Ploss$ regarding the parameter s is known, then the computation of $\Delta Ploss$ requires the calculation of the integral (2.97). This integral can be calculated in an approximate way using a trapezoidal approximation that leads to (2.98).

$$P_t(s) = s.P_t \tag{2.96}$$

$$\Delta Ploss = \int_{s=0}^{s=1} \frac{\partial Ploss}{\partial s}.ds \tag{2.97}$$

$$\Delta Ploss \cong \Delta s.\left.\frac{\partial Ploss}{\partial s}\right|_{s=0,5} \tag{2.98}$$

As one adds new transactions, there are variations of voltage magnitude and phases. Under these conditions, the derivative of $\Delta Ploss$ regarding s can be expressed using partial derivatives of s regarding voltage magnitudes and phases (2.99).

$$\frac{\partial Ploss}{\partial s}=\frac{\partial Ploss}{\partial \theta_1}.\frac{\partial \theta_1(s)}{\partial s}+\frac{\partial Ploss}{\partial V_1}.\frac{\partial V_1(s)}{\partial s}+\cdots\frac{\partial Ploss}{\partial \theta_n}.\frac{\partial \theta_n(s)}{\partial s}+\frac{\partial Ploss}{\partial V_n}.\frac{\partial V_n(s)}{\partial s} \quad (2.99)$$

Let us assume that X (2.100) is a vector that includes the voltage magnitudes and phases in each node. Let also assume that (2.101) represents the derivatives of $Ploss$ regarding X. Then, $\Delta Ploss$ is given by (2.102).

$$X=[\theta_1, V_1, \cdots, \theta_n, V_n] \quad (2.100)$$

$$\frac{\partial Ploss}{\partial X}X=\left[\frac{\partial Ploss}{\partial \theta_1},\frac{\partial Ploss}{\partial V_1},\cdots,\frac{\partial Ploss}{\partial \theta_n},\frac{\partial Ploss}{\partial V_n}\right] \quad (2.101)$$

$$\Delta Ploss \cong \Delta s.\frac{\partial Ploss}{\partial X}.\begin{bmatrix}\frac{\partial \theta_1(s)}{\partial s}\\ \frac{\partial V_1(s)}{\partial s}\\ \vdots\end{bmatrix} \quad (2.102)$$

The derivatives of voltage magnitudes and phases regarding the parameter s can be obtained after running a power flow study using the Newton-Raphson, while admitting that injected powers are also expressed in terms of that parameter. Under these conditions, one can use the equations (2.103) in which $J^{-1}(\theta, V)$ represents the inverse of the Jacobean matrix evaluated for s=0,5.

$$\begin{bmatrix}\frac{\partial \theta_1(s)}{\partial s}\\ \frac{\partial V_1(s)}{\partial s}\\ \vdots\end{bmatrix}=J^{-1}(\theta, V).\begin{bmatrix}\frac{\partial P_1(s)}{\partial s}\\ \frac{\partial Q_1(s)}{\partial s}\\ \vdots\end{bmatrix} \quad (2.103)$$

This expression can be substituted in (2.102), leading to an expression for $\Delta Ploss$ due to a transaction involving a power Δs (2.104).

$$\Delta Ploss \cong \Delta s.\frac{\partial Ploss}{\partial X}.J^{-1}(\theta, V).\begin{bmatrix}\frac{\partial P_1(s)}{\partial s}\\ \frac{\partial Q_1(s)}{\partial s}\\ \vdots\end{bmatrix} \quad (2.104)$$

Finally, admitting that each injected power depends of the parameter s and it expressed as the multiplication of s by the global power involved in that transaction, it is possible to rewrite (2.104) obtaining (2.105). In this expression, the derivatives of active and reactive injected powers are substituted by a sum of vectors each one of them associated to one transaction.

$$\Delta Ploss \cong \Delta s.\frac{\partial Ploss}{\partial X}.J^{-1}(\theta,V).\left(\begin{bmatrix} \cdots \\ P_{t=1} \\ \cdots \\ -P_{t=1} \\ \cdots \end{bmatrix} + \cdots + \begin{bmatrix} \cdots \\ \cdots \\ P_{t=1} \\ \cdots \\ -P_{t=1} \end{bmatrix}\right) \tag{2.105}$$

This means that the vector associated to a transaction involving node i as generation node and node j as demand node will have zero values is all its lines except on the lines related with the active injected power for node i (in which there is the generation involved in that transaction) and in line j (in which there the symmetric of the active load in that node). Therefore, the active losses allocated to transaction t=1, for instance, will be given by (2.106).

$$\Delta Ploss \cong \Delta s.\frac{\partial Ploss}{\partial X}.J^{-1}(\theta,V).\begin{bmatrix} \cdots \\ P_{t=1} \\ \cdots \\ -P_{t=1} \\ \cdots \end{bmatrix} = \Delta s.\frac{\partial Ploss}{\partial X}.J^{-1}(\theta,V).\begin{bmatrix} \cdots \\ 1 \\ \cdots \\ -1 \\ \cdots \end{bmatrix}.P_t = \eta_t.P_t \tag{2.106}$$

This approach requires running a power flow study to get an operation point of the system in which we consider half of the powers involved in all transactions. The reason to consider all of these powers comes from the trapezoidal approximation in (2.98) adopted to compute the integral (2.97). In that operation point they will be computed several derivatives in (2.106) as well as the Jacobean matrix and its inverse. Finally, the relation (2.106) leads to the calculation of the loss allocation coefficient η_t regarding transaction t, in terms of the power involved in that transaction.

2.4. Methodologies Based on the DC Model

2.4.1. Formulation of the Basic Problem

Rivier et al (1990), (1993), (1994) developed the JUANAC model to perform dispatch studies of a generation system including hydro and thermal stations integrating a model of the grid based on the DC model. In brief way, the optimisation problem can be formulated as (2.107) to (2.112).

$$\min z = \sum c_k .Pg_k + G.\sum PNS_k \tag{2.107}$$

$$\sum Pg_k + \sum PNS_k = \sum Pl_k \tag{2.108}$$

$$Pg_k^{min} \le Pg_k \le Pg_k^{max} \tag{2.109}$$

$$PNS_k \le Pl_k \tag{2.110}$$

$$\sum a_{bk}.(Pg_k + PNS_k - Pl_k) \le P_b^{max} \tag{2.111}$$

$$\sum a_{bk}.(Pg_k + PNS_k - Pl_k) \ge P_b^{min} \tag{2.112}$$

In this formulation:

- c_k is the generation cost of the generator connected to node k and G is the penalty specified for Power Not Supplied, PNS;
- Pg_k represents the generation in node k;
- the equation (2.108) aims at balancing the generation and the demand considering that Pl_k is the load connected to node;
- constraints (2.109) e (2.10) impose limits to the generated power in each machine and to the PNS in each node k;
- constraints (2.111) and (2.112) impose limits to the branch power flows in each branch b of the grid. In these constraints, the coefficients a_{bk} are the sensitivity coefficients of the DC Model expressing the relation of the power flow in branch b and the injected power in node k.

This formulation corresponds to a linear optimisation problem that can be efficiently solved by the Simplex Method. In any case, this formulation does not include any estimate of active losses. If such an estimate was not included, the geographic dispersion of nodal marginal prices would only result from branch congestion situations.

The analysed references indicate two algorithms to obtain loss estimates in the grid in to include it in the above described model. These two algorithms will be detailed in the next section given their relevance to obtain adequate estimates of nodal marginal prices.

2.4.2. Integration of an Estimate of Active Losses – Model A

This model approximates the active losses in each branch i-j of the grid by expression (2.5) that results of the exact expression of branch active losses assuming that voltage magnitudes are equal to 1,0 pu.

Due to the integration of an estimate of active losses, the above optimisation problem (2.107) to (2.112) has to be solved a number of times in an iterative way. At the end of each of these solutions it is computed an estimate of active losses in each branch using (2.5). In this algorithm, at the end of each iteration, corresponding to the solution of a dispatch problem – half of the losses in each branch are added to the load in the extreme buses of that branch. This change of the loads requires solving a new dispatch problem in order to change the generation to accommodate load changes. The experience of the authors indicates that this iterative process converges in 2 to 4 iterations.

2.4.3. Integration of an Estimate of Active Losses – Model B

Any way of solving integrating an estimate of losses consists of running a first dispatch problem in the absence of any estimate of active losses. Afterwards, the operation point obtained this way is adopted as linearisation point to build a linearised expression for the active losses in each branch. This linearaised expression depends on the phases in each node. This expression is used to modify the balance generation/load for each node, so that we get a modified linear optimisation dispatch problem. In this formulation the voltage phases are decision variables of the problem, differently from other formulation in which these variables are not explicitly considered.

Let us consider that when one of those dispatch problems finishes, we obtain the voltage phases θ_k in each node k. Using these values, we can build the linearised expression to approximate active losses in branch i-j considering the tangent line to the curve associated to (2.5) in the current linearisation point. This approximation leads to the expressions (2.8), (2.9) and (2.10) already presented in section 2.2 of this report.

As in model A, half of the active losses in each branch is added to the load in each of the extreme buses of that branch, leading to the formulation (2.113) to (2.118).

$$\min z = \sum c_k.Pg_k + G.\sum PNS_k \tag{2.113}$$

$$\sum Pg_k + \sum PNS_k - \sum_{\text{branches}} CL_{ij}^2 \theta_{ij} = \sum Pl_k + \sum_{\text{branches}} CL_{ij}^1 \tag{2.114}$$

$$Pg_k + PNS_k - \sum_j BDC_{kj}.\theta_j - \sum_j \frac{CL_{kj}^2}{2}.\theta_{kj} = Pl_k + \sum_j \frac{CL_{kj}^1}{2} \tag{2.115}$$

$$Pg_k^{min} \le Pg_k \le Pg_k^{max} \tag{2.116}$$

$$PNS_k \le Pl_k \tag{2.117}$$

$$P_{ij}^{min} \le \frac{\theta_{ij}}{x_{ij}} \le P_{ij}^{max} \tag{2.118}$$

In this formulation:
- (2.114) represents the global balance equation of generated powers, demand and power not supplied;
- (2.115) represents the nodal balance equations, formulated with elements of the B matrix of the DC model;
- constraints (2.116) and (2.117) impose limits to the generated powers and power not supplied in each node;
- constraints (2.116) impose limits to branch active flows. The active flows are computed with the voltage phases in the extreme buses of each branch since these are decision variables of this optimisation problem.

When running this problem for the first time, voltage phases are zero so that this model is in fact equivalent to the one used in Model A. Once the first dispatch study is completed, we obtain a first set of voltage phases that can be used to compute the coefficients (2.9) and (2.10) for each branch. Once these coefficients are computed for all branches when can include the linearised loss expressions in problem (2.113) to (2.118) in order to run a new dispatch problem to update voltage phases.

2.4.4. Computation of Marginal Prices and Marginal Loss Coefficients

Nodal marginal prices are computed using the general expression (2.119) as subproducts of the solution of the above described problems. According to this expression, the marginal price in node k corresponds to the impact on the cost function, z, if there is a change in the load in node k, Pl_k. None of the two described models include information about reactive flows, problems related with voltage regulation, uncertainty related to the nodal injected powers or contingencies. In this sense we are obtaining nodal marginal prices of active energy for a single configuration of the system for which a load scenario was specified. When using Models A or B we obtain particular expressions for the nodal marginal prices corresponding respectively to (2.120) and (2.121).

$$\rho_k = \frac{\partial z}{\partial Pl_k} \tag{2.119}$$

$$\rho_{k,\text{Model A}} = \frac{\partial z}{\partial Pl_k} = \gamma.\left(1{,}0 + \frac{\partial Ploss}{\partial Pl_k}\right) + \sum_{\substack{\text{all} \\ \text{branches}}} \mu_{ij}.\frac{\partial P_{ij}}{\partial Pl_k} + \sigma_k \tag{2.120}$$

$$\rho_{k,\text{Model B}} = \frac{\partial z}{\partial Pl_k} = \gamma_k + \sigma_k \tag{2.121}$$

In these expressions:

- γ represents the dual variable to balance equation (2.108);
- μ_{ij} represents the dual variable of branch flow maximum (2.111) or minimum (2.112) constraint for branch i-j that is on one of these;
- σ_k represents the dual variable of the constraint imposing a limit to the power not supplied in node k (2.110) or (2.117);
- γ_k represents the dual variable of the balance equation regarding node k (2.115);
- P_{ij} is the active power flow in branch i-j. Its derivative regarding the load in node k, Pl_k, corresponds to the symmetric of the sensitivity coefficient of the active flow in branch i-j regarding the injected power in node k;
- finally, the derivative of active losses in the whole, $Ploss$, regarding the active load in node k, Pl_k, is given by (2.124). In this expression, Z_{ik} and Z_{jk} represent the elements of line i/column k of the inverse of the B matrix of the DC Model, once we eliminated one line and one column. This expression was obtained admitting that the losses in the whole grid correspond to the addition of the active losses in all its branches (2.122). Afterwards, the active losses in branch i-j is approximately given by (2.5) so that (2.123) already includes the derivative of (2.5) regarding Pl_k. Finally, in the DC Model the derivative of voltage phases regarding the injected power in one node corresponds to one element of the Z matrix corresponding to the inverse of the B matrix already referred. According to (2.123), the derivatives are computed regarding the active load in node k. That is why they correspond to symmetric of elements of Z matrix just referred. This reasoning finally leads to (2.124).

$$\frac{\partial Ploss}{\partial Pl_k} = \sum_{\text{all branches}} \frac{\partial Ploss_{ij}}{\partial Pl_k} \tag{2.122}$$

$$\frac{\partial Ploss}{\partial Pl_k} = \sum_{\text{all branches}} 2.g_{ij}.\sin(\theta_{ij}).\left(\frac{\partial \theta_i}{\partial Pl_k} - \frac{\partial \theta_j}{\partial Pl_k}\right) \tag{2.123}$$

$$\frac{\partial Ploss}{\partial Pl_k} = \sum_{\text{all branches}} 2.g_{ij}.\sin(\theta_{ij}).(-Z_{ik} + Z_{jk}) \tag{2.124}$$

Under these conditions, and according to Model A, the marginal nodal loss coefficients reflecting the impact of change in the load in node k are given by (2.125). The nodal marginal loss coefficients regarding a change in the generation in node k corresponds to the symmetric of (2.125).

$$K_{k,load} = \frac{\partial Ploss}{\partial Pl_k} \tag{2.125}$$

2.4.5. Comparison Between Models A and B

Having described these two models to compute nodal marginal prices, it is now possible to write some comments on them:

- Model A includes a single balance equation between generations, load and power not supplies (2.108) and the power flow constraints are written in terms of the DC Model sensitivity coefficients. The computation of these sensitivity coefficients require building and inverting the B matrix of the DC model. The inversion of this matrix requires selecting a bus for reference of voltage phases;
- under these conditions, when we are evaluating the impact of the marginal variation of active losses due to a marginal variation of active load in node k, it is necessary to use the concept of a slack node. In Model A the slack node coincides with the node selected for reference of the voltage phases. This situation is taken in account when writing expression (2.124) to use to evaluate the marginal variation of active losses in node k. This expression requires the calculation of the derivative of the active losses in the grid regarding the active load in node k and this calculation is performed using elements of the inverse of the B matrix of the DC model. These elements depend on the line/column of the B matrix that was eliminated that is, they depend on the bus selected for reference;
- this means that there two important concepts – reference node and slack node. In this case, the slack node corresponds to the node in which it will generate power to compensate the marginal variation of active losses. In a dispatch problem this node would correspond to the marginal node of the system. If the node to which the marginal generator is not selected for reference (that according to the mathematical formulation usually coincides with the slack node) there will be a difference between the in which marginal variations of active losses should be compensated (node to which the marginal generator is connected to) and the node in which that variation will in fact be considered (reference node). This situation explains the dependence of the loss coefficients given by (2.124) regarding the reference and slack node;
- to obtain correct nodal marginal prices and loss marginal coefficients it is therefore important to select the reference+slack node so that it coincides with the node to which it is connected the marginal generator. This is not always easy to guarantee. In fact, the marginal generator is usually very volatile since it depends on the operation conditions of the grid and its topology, on the load level, on the generators and branches in service, etc. Apart from this, the marginal generator function can be assigned to more than one generator. In any case, obtaining correct marginal loss coefficients using (2.125) implies a good selection of the reference+slack bus, in the sense one should make it coincide with the node it is connected the marginal generator;
- Model B includes as many balance equations (2.115) as the number of nodes of the grid. Apart from this, the active flow branch constraints are directly written in terms of

voltage phases. This turns it unnecessary to invert the B matrix of the DC Model. In this sense, the nodal marginal prices and the marginal loss coefficients do not depend on the bus selected for reference of the voltage phases. This is an important advantage of Model B to be stressed;

- on the contrary, in Model B the nodal marginal prices are given by (2.121). This means that the components of these prices, and namely the loss term, are obtained separately. This is a disadvantage since several applications require the knowledge of the congestion and loss components separately.

The aspects related with the dependence of the terms of the nodal marginal price on bus k (2.120) regarding the bus selected for reference+slack are analysed in detail in Rivier et al (1993).

According to Model A it is necessary to select a node for reference+slack. This means that any partial derivative will measure the sensitivity of a given function regarding a set of independent variables while maintaining some other constant. In this case, the generations in all generators except the one connected to the reference+slack bus will be kept constant. Physically, this means that the generator connected to the reference+slack node will change its output to maintain the balance in the system. In order to turn this dependence more visible the expression (2.120) was rewritten including now the index rs and admitting that the dual variables regarding power not supplied are zero (2.126).

$$\rho_{k,\text{Model A}} = \gamma_{rs} \cdot \left(1{,}0 + \frac{\partial Ploss}{\partial Pl_k\big|_{rs}} \right) + \sum_{\substack{\text{all} \\ \text{branches}}} \mu_{ij} \cdot \frac{\partial P_{ij}}{\partial Pl_k\big|_{rs}} \tag{2.126}$$

Under these conditions, the nodal price in node rs (reference+slack node) is given by (2.127) since the derivative of active losses regarding the active load in node rs, Pl_{rs}, is zero and the sensitivity coefficients of any branch flow regarding the node rs are also zero.

$$\rho_{rs,\text{Model A}} = \gamma_{rs} \cdot \left(1{,}0 + \frac{\partial Ploss}{\partial Pl_{rs}\big|_{rs}} \right) + \sum_{\substack{\text{all} \\ \text{branches}}} \mu_{ij} \cdot \frac{\partial P_{ij}}{\partial Pl_{rs}\big|_{rs}} = \gamma_{rs} \tag{2.127}$$

Therefore, the nodal marginal in any node k can be expressed in terms of the marginal price in node rs (2.128) or, in a more general way, the marginal price in any node k_1 can be expressed in terms of the marginal price in another node k_2 using (2.129).

$$\rho_{k,\text{Model A}} = \rho_{rs,\text{Model A}} \cdot \left(1{,}0 + \frac{\partial Ploss}{\partial Pl_k\big|_{rs}} \right) + \sum_{\substack{\text{all} \\ \text{branches}}} \mu_{ij} \cdot \frac{\partial P_{ij}}{\partial Pl_k\big|_{rs}} \tag{2.128}$$

$$\rho_{k1,\text{Model A}} = \gamma_{k2,\text{Model A}} \cdot \left(1{,}0 + \frac{\partial Ploss}{\partial Pl_{k1}\big|_{k2}} \right) + \sum_{\substack{\text{all} \\ \text{branches}}} \mu_{ij} \cdot \frac{\partial P_{ij}}{\partial Pl_{k1}\big|_{k2}} \tag{2.129}$$

The decomposition of the nodal marginal prices can be obtained if one defines an average marginal price that, from a regulatory point of view, is considered to represent the system and regarding which the congestion and loss components can be computed.

This can correspond to the definition of a center of loads regarding which, according to Rivier et al (1993), nodal marginal prices can be obtained. (2.130). This expression assumes that that the γ component should depend on the distribution of nodal marginal prices so that, in tariff terms, the generators and loads are not favoured with artificial increases or decreases of the amounts to receive or to pay

$$\gamma = \frac{\sum_k \rho_k.Pl_k + \sum_k \rho_k.Pg_k}{\sum_k Pl_k + \sum_k Pg_k} \tag{2.130}$$

The use of a concept as this one would lead to the computation of the loss component of nodal marginal prices in an independent way of the selected node for reference+slack, provided that there were no congestion situations. This means that the last terms in expressions (2.126), (2.128) and (2.129) would all be zero. If there was one of them that was not zero, then we would only be able to obtain in an aggregated way the loss and congestion components of the nodal marginal prices.

Under these conditions, having in mind all the comments included in the first part of this section and in order to get the loss component separately obtaining simultaneously more realistic results, it is more adequate to use Model A already described. This Model should be used carefully since it is important to adequately select the reference+slack bus, that, this bus should coincide, if possible, with the node where it is connected the marginal system generator.

3. Method Adopted to Allocate Losses

Costa et al (2004) describe a method to allocate active losses in distribution networks specially designed for grid where there are embedded generation. The authors explain that active losses are difficult to allocate since they have a non linear nature and change with voltage variations. The allocation method proposed in this reference was designed to accomplish several principles allocation methods should have: objectivity, be easily understood, be based on real data of the grid, be economically efficient and avoid cross subsidisation and be able to recover to global amount of losses in order to turn it unnecessary the use of revenue reconciliation methods.

In order to accomplish these objectives the authors designed a loss allocation method that includes three phases: consumer's loss allocation, loss allocation to embedded generators and finally the allocation of voltage-related loss variations.

3.1. Phase 1 – Consumer's Loss Allocation

In the first place, it is run an AC power flow study to evaluate the losses in the grid without embedded generators. The results obtained in this situation will be designated as Base Case. These results include active and reactive branch flows and real and imaginary parts of branch currents defined by (3.1). The "Downstream Looking Method" described in Bialek (1996-b), and summarised in section 3.4), can now be applied to the branch currents in order to separate the contribution of each load j to the real and imaginary parts of these currents (3.2). In this expression R_i is the resistance of branch i and Z is number of consumers.

$$\underline{I}_i^o = I_i^{op} - jI_i^{oq} \tag{3.1}$$

$$Ploss_i = R_i\left(\left(\sum_{j=1}^{Z} I_{ji}^{op}\right)^2 + \left(\sum_{j=1}^{Z} I_{ji}^{oq}\right)^2\right) \tag{3.2}$$

Each consumer j contributes to determine two sets of terms in component i of the grid. The first set includes terms as (3.3) and (3.4).

$$\left(I_{ji}^{op}\right)^2 \tag{3.3}$$

$$\left(I_{ji}^{oq}\right)^2 \tag{3.4}$$

The second set includes cross terms due to consumer j and any other consumer k as (3.5) and (3.6).

$$2.\sum_{\substack{k=1 \\ k\neq i}}^{Z} I_{ji}^{op}.I_{ki}^{op} \tag{3.5}$$

$$2.\sum_{\substack{k=1 \\ k\neq i}}^{Z} I_{ji}^{oq}.I_{ki}^{oq} \tag{3.6}$$

Terms (3.3) and (3.4) are only due to consumer j and therefore are inherently allocated. However, terms (3.5) and (3.6) must be allocated since they are crossed terms expressing the interaction between consumer j and any other consumer k. In this reference, the authors adopted a quadratic approach since this seems to be more adequate as active losses also depend in a quadratic way on currents. Regarding component i of the grid, the quadratic allocation of the losses of the crossed terms together with the terms that are inherently allocated to consumer j leads to (3.7). The global values of the losses to be paid by consumer j are the sum of the loss allocation obtained for all the components i in the grid.

$$\begin{aligned} Ploss_i^j = R_i \Bigg[& \left(I_{ji}^{op}\right)^2 + 2\sum_{\substack{k=1 \\ k\neq j}}^{Z} I_{ji}^{op} I_{ki}^{op} \frac{\left(I_{ji}^{op}\right)^2}{\left(I_{ji}^{op}\right)^2 + \left(I_{ki}^{op}\right)^2} + \\ & + \left(I_{ji}^{oq}\right)^2 + 2\sum_{\substack{k=1 \\ k\neq j}}^{Z} I_{ji}^{oq} I_{ki}^{oq} \frac{\left(I_{ji}^{oq}\right)^2}{\left(I_{ji}^{oq}\right)^2 + \left(I_{ki}^{oq}\right)^2} \Bigg] \end{aligned} \tag{3.7}$$

3.2. Phase 2 – Loss Allocation to Embedded Generators

The second phase starts with a second power flow study considering the presence of embedded generators. This power flow computes the active and reactive branch flows as well as the real and imaginary parts of branch currents. We will now use the "Downstream Looking Algorithm", summarised in section 3.4, to compute the contributions of each consumer j to the real and imaginary currents, I_{ji}^{1p} and I_{ji}^{1q}, in each element i of the grid. Having computed these currents, we can now compute their variations using (3.8) and (3.9).

$$\Delta I_{ji}^{p} = I_{ji}^{1p} - I_{ji}^{op} \tag{3.8}$$

$$\Delta I_{ji}^{q} = I_{ji}^{1q} - I_{ji}^{oq} \tag{3.9}$$

These variations can now be allocated to the embedded generators. To perform this allocation, it is necessary to compute some auxiliary quantities required to apply the "Upstream Looking Algorithm" as detailed in Bialek (1996-b), and summarised in section 3.4. These quantities are:

- A_j^{pk} - contribution of generator k to the real part of the current of consumer j;
- A_j^{qk} - contribution of generator k to the imaginary part of the current of consumer j;
- B_i^{pk} - contribution in the inverse direction of generator k to the real component of the current in branch i;

- B_i^{qk} - contribution in the inverse direction of generator k to the imaginary component of the current in branch i;
- C_i^{pk} - contribution in the direct direction of generator k to the real component of the current in branch i;
- C_i^{qk} - contribution in the direct direction of generator k to the imaginary component of the current in branch i.

These definitions mean that it is necessary to define positive directions for the flows. By definition, the way the current flow in each component in the base case corresponds to the positive direction. In some cases there are flows that are zero in the base case and that only assume non zero values after considering the embedded generators. In these cases, the positive direction is only defined after considering the embedded generators.

Now we can proceed to perform the allocation of the current variations given by (3.8) and (3.9). If one of these variations is negative, then the variation of the real or imaginary part of the current in component i, due to consumer j is allocated to generator k according to (3.10) and (3.11).

$$\begin{cases} \Delta I_{ji}^{pk} = \dfrac{A_j^{pk}}{\sum\limits_{l\in\gamma} A_j^{pl}}.\Delta I_{ji}^{p} \quad \text{if } I_{ji}^{op} > 0 \text{ and } k \in \gamma \\ \Delta I_{ji}^{qk} = \dfrac{A_j^{qk}}{\sum\limits_{l\in\gamma} A_j^{ql}}.\Delta I_{ji}^{q} \quad \text{if } I_{ji}^{oq} > 0 \text{ and } k \in \gamma \end{cases} \tag{3.10}$$

$$\begin{cases} \Delta I_{ji}^{pk} = \dfrac{B_i^{pk}}{\sum\limits_{l\in\rho} B_i^{pl}}.\Delta I_{ji}^{p} \quad \text{if } I_{ji}^{op} = 0 \text{ and } k \in \rho \\ \Delta I_{ji}^{qk} = \dfrac{B_i^{qk}}{\sum\limits_{l\in\rho} B_i^{ql}}.\Delta I_{ji}^{q} \quad \text{if } I_{ji}^{oq} = 0 \text{ and } k \in \rho \end{cases} \tag{3.11}$$

If that variation is positive, then the allocation is performed using (3.12).

$$\begin{cases} \Delta I_{ji}^{pk} = \dfrac{C_i^{pk}}{\sum\limits_{l\in\beta} C_i^{pl}}.\Delta I_{ji}^{p} \quad \text{if } I_{ji}^{op} \geq 0 \text{ and } k \in \beta \\ \Delta I_{ji}^{qk} = \dfrac{C_i^{qk}}{\sum\limits_{l\in\beta} C_i^{ql}}.\Delta I_{ji}^{q} \quad \text{if } I_{ji}^{oq} \succ 0 \text{ and } k \in \beta \end{cases} \tag{3.12}$$

In expressions (3.10), (3.11) and (3.12), the sets γ, ρ and β are defined as:

- γ - set of embedded generators that contribute to the current in consumer j without contributing for the same component of the current in branch i in the direct direction;
- ρ - set of embedded generators that contribute in the inverse direction to the current in branch i and simultaneously to the same component of the current of consumer j;
- β - set of embedded generators that contribute in the direct direction to the current in branch i and simultaneously to the same component of the current of consumer j.

Once these variations are computed, it is possible to calculate the global variations of the real and imaginary currents allocated to each generator (3.13) and (3.14) in each component k.

$$\Delta I_i^{pk} = \sum_{j=1}^{Z} \Delta I_{ji}^{pk} \tag{3.13}$$

$$\Delta I_i^{qk} = \sum_{j=1}^{Z} \Delta I_{ji}^{qk} \tag{3.14}$$

It is now possible to allocate the variations of losses in each component k to the embedded generators. To perform this allocation it is important to notice that the losses before connecting the embedded generators are given by (3.2). After connecting the embedded generators the losses are given by (3.15). In this expression, H represents the number of embedded generators in the grid.

$$Ploss_i = R_i \left(\left(I_i^{op} + \sum_{k=1}^{H} \Delta I_i^{pk} \right)^2 + \left(I_i^{oq} + \sum_{k=1}^{H} \Delta I_i^{qk} \right)^2 \right) \tag{3.15}$$

Using the same proportional sharing algorithm as in Phase 1 and adopting the same quadratic allocation scheme it is possible to allocate the loss variations in each component i to each embedded generator k using (3.16).

$$\begin{aligned} Ploss_i^k = R_i \Bigg[& \left(\Delta I_i^{pk}\right)^2 + 2.I_i^{op}.\Delta I_i^{pk} + \\ & + 2.\sum_{\substack{l=1 \\ l\neq k}}^{H} \left(\Delta I_i^{pk}.I_i^{pl}.\frac{\left(\Delta I_i^{pk}\right)^2}{\left(\Delta I_i^{pk}\right)^2 + \left(\Delta I_i^{pl}\right)^2} \right) + \\ & + \left(\Delta I_i^{qk}\right)^2 + 2.I_i^{oq}.\Delta I_i^{qk} + \\ & + 2.\sum_{\substack{l=1 \\ l\neq k}}^{H} \left(\Delta I_i^{qk}.I_i^{ql}.\frac{\left(\Delta I_i^{qk}\right)^2}{\left(\Delta I_i^{qk}\right)^2 + \left(\Delta I_i^{ql}\right)^2} \right) \Bigg] \end{aligned} \tag{3.16}$$

Once this procedure is used for all N grid components, the global loss variations allocated to each generator k are given by (3.17).

$$Ploss^{k} = \sum_{i=1}^{N} Ploss_{i}^{k} \qquad (3.17)$$

3.3. Phase 3 – Allocation of Voltage-Related Loss Variations

Once the embedded generators are connected to the grid, the voltage profile changes, as well as the losses in the components of the grid. These loss variations due to voltage changes must now be allocated. These voltage related loss variations are given by (3.18). In this expression, $Ploss^{1}$ and $Ploss^{o}$ are the active losses considering the embedded generators are connected to the grid and the active losses considering the embedded generators are not connected and $Ploss^{\alpha}$ are the active losses allocated to the generators in the phases 1 and 2.

$$Ploss^{volt} = \left(Ploss^{1} - Ploss^{o}\right) - Ploss^{\alpha} \qquad (3.18)$$

Although some more involving techniques could be adopted, the allocation of this voltage related loss variations to the generators can be performed in a proportional way regarding the apparent power of each generator. This is translated by expression (3.19) in what would correspond to the application of the postage stamp principle regarding these loss variations.

$$Ploss_{m}^{volt} = \frac{S_m}{\sum_{j=1}^{H} S_j} . Ploss^{volt} \qquad (3.19)$$

3.4. Downstream and Upstream Looking Algorithms

Reference Bialek (1996.b) describes the Downstream and the Upstream looking algorithms to trace electricity, that is, to evaluate how electricity from generators is distributed to loads using transmission lines. The two referred algorithms work on ideal loss less networks, that is, they assume that the powers in the two extremes of a line are equal. If the power flow results for real network are provided, it is possible to estimate the flows in a loss less network by getting the average values of the power in both extremities of each line. Once these results are obtained, the Downstream Looking Algorithm will trace electricity considering the nodal balance of outflows while the Upstream Looking Algorithm will adopt the nodal balance of inflows.

Upstream Looking Algorithm

This algorithm uses expression (3.20) to represent the power flowing through node i. In this expression, α_i^u represents the set of nodes supplying directly node i, P_{ji} is flow in line i-j towards node i and Pg_i is the generation in node i. This expression can be rearranged by substituting the flows in lines ji by terms as $c_{ji}.P_j$. This substitution leads to expression (3.21).

$$P_i = \sum_{j \in \alpha_i^{(u)}} \left|P_{ji}\right| + Pg_i \qquad (3.20)$$

$$P_i = \sum_{j \in \alpha_i^{(u)}} c_{ji}.P_j + Pg_i \qquad (3.21)$$

Expression (3.21) can now be changed in order to get nodal generations expression in terms of a matrix A^u and a vector P of nodal through flows (3.22). Matrix A^u includes information about the upstream distribution flows. The ij element of this matrix is given by (3.23).

$$Pg = A^u.P \qquad (3.22)$$

$$A_{ij}^u = \begin{cases} 1 & \text{for } i = j \\ -c_{ji} = -\dfrac{|Pji|}{P_j} & \text{for } j \in \alpha_i^{(u)} \\ 0 & \text{other cases} \end{cases} \qquad (3.23)$$

Once this matrix is obtained, it is possible to obtain the power outflow regarding node i in line i-l using expression (3.24). This expression was established admitting that the flows entering any node are distributed proportionally between the outflows, in what is called the proportional sharing principle. In this expression n is the number of nodes, P_{il} is the flow in line il from node i to l and P_i is the is the total flow through node i. This expression shows that it is possible to express the nodal outflows in terms of the generated power in each node.

$$|P_{il}| = \frac{|P_{il}|}{P_i}.P_i = \frac{|P_{il}|}{P_i}.\sum_{k=1}^{n}\left[A_{ik}^u\right]^{-1}.Pg_k \qquad (3.24)$$

Downstream Looking Algorithm

The downstream looking algorithm tries to obtain an expression similar to (3.24) but now considering the demand in each node i. To get this expression this algorithm uses expression (3.25) in terms of the line outflows and the demand in node i. In a similar way regarding the upstream algorithm, this expression can be rewritten using the c_{li} coefficients defined as the quotient of $|P_{li}|$ and P_l.

$$P_i = \sum_{l \in \alpha_i^{(u)}} |P_{li}| + Pl_i \qquad (3.24)$$

$$P_i = \sum_{j \in \alpha_i^{(u)}} c_{ji}.P_j + Pg_i \qquad (3.25)$$

In an analogue way, it is built a matrix A^d whose element il is given by (3.26) expressing the relation between the vector of nodal demands and the vector of nodal through flows (3.27). In these expressions, α_i^d is the set of nodes directly supplied by node i.

$$A_{il}^d = \begin{cases} 1 & \text{for } i = l \\ -c_{li} = -\dfrac{|P_{li}|}{P_l} & \text{for } l \in \alpha_i^{(d)} \\ 0 & \text{other cases} \end{cases} \qquad (3.26)$$

$$Pl = A^d.P \qquad (3.27)$$

Once this matrix is built, the inflow to node i regarding line i-j can be obtained using expression (3.28). This expression shows that it is possible to express the nodal inflows in terms of the demand in each node.

$$|Pj| = \frac{|P_{ij}|}{P_i}.P_i = \frac{|P_{ij}|}{P_i}.\sum_{k=1}^{n}\left[A_{ik}^{d}\right]^{-1}.Pl_k \qquad (3.28)$$

4. Simulations

4.1. Data

In order to illustrate the loss allocation approach presented in section 3, we used the “Study-Case LV Network” whose characteristics are detailed in the document Study-Case LV-Network.pdf by Stavros Papathanassiou. The LV network single line diagram is presented in Figure 4.1.

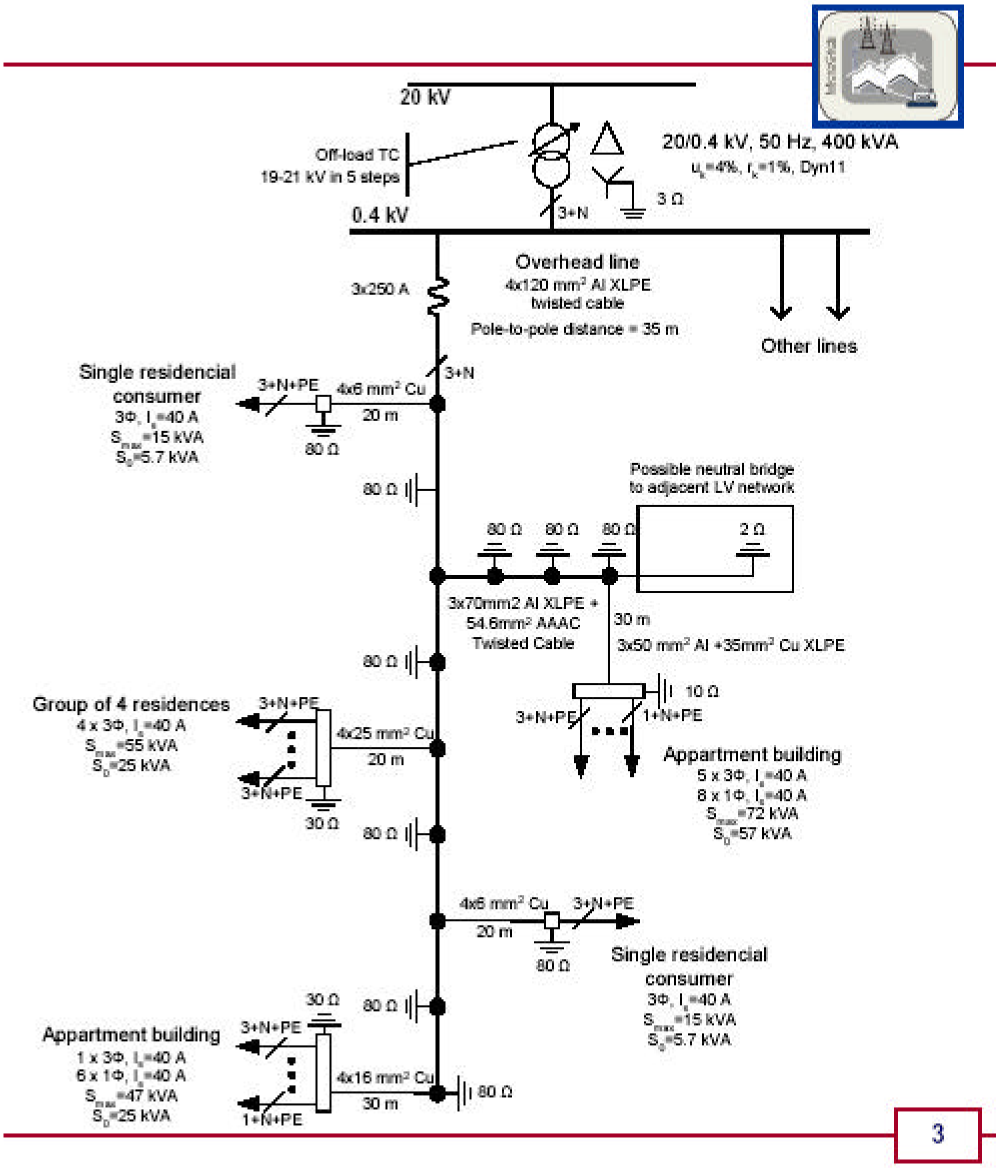

Figure 4.1 – Single line diagram of the LV network study case.

Using this data we built the LV network represented in Figure 4.2 considering the microsources, the loads and the node numbering adopted in the simulations to be described. For this network, Table 4.1 indicates the branch characteristics

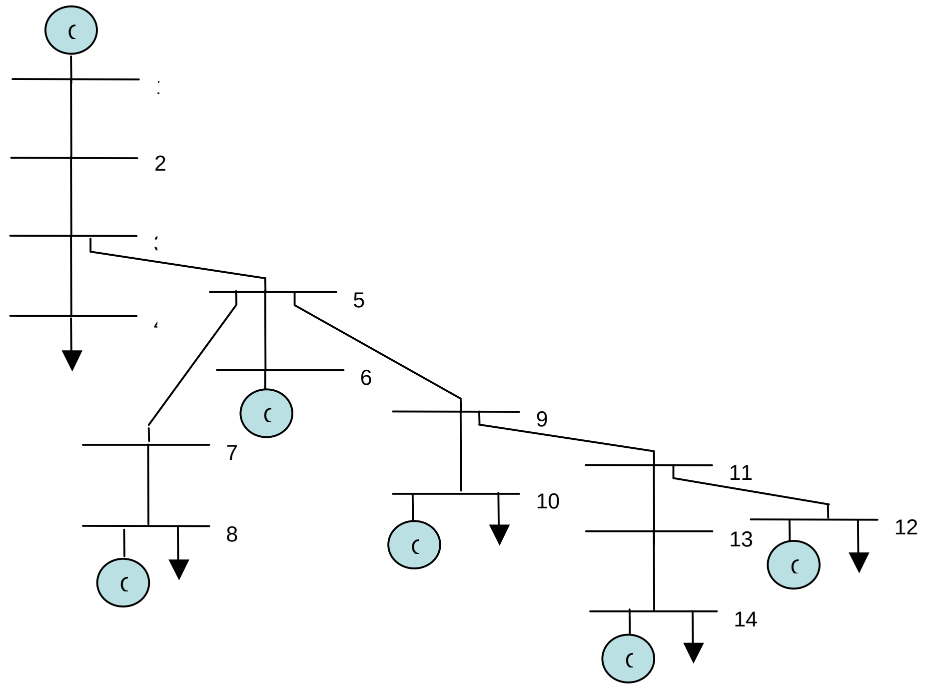

Figure 4.2 – Single line diagram considering the microsources, loads and node numbering.

Table 4.1 – Branch characteristics regarding a 0,4 MVA basis.

branch	Node i	Node j	Rij [pu]	Xij [pu]	Yshij/2 [pu]
1	1	2	0	0,04	0
2	2	3	0,02485	0,007263	0
3	3	4	0,1705	0,0047	0
4	3	5	0,0497	0,014525	0
5	5	6	0,1035	0,006375	0
6	5	7	0,130463	0,02625	0
7	7	8	0,03465	0,005775	0
8	5	9	0,0497	0,014525	0
9	9	10	0,0435	0,00415	0
10	9	11	0,0497	0,014525	0
11	11	12	0,1705	0,0047	0
12	11	13	0,0497	0,014525	0
13	13	14	0,1035	0,006375	0

Using this information we ran the loss allocation algorithm described in section 3 in two cases corresponding to two different sets of generation values from the microsources and from the main MV grid. These two simulations will be described in the next two sections. It should be noted that the method described in Section 3 can be used to allocate losses to consumers and then to allocate avoided losses to the microsources in the grid. In our simulations, we considered that the microsources were connected to the grid from the beginning and therefore we are simply interested in allocating losses to consumers considering microsurces connected to the grid.

4.2. Case 1

For Case 1, Tables 4.3 and 4.4 present the specified values and the results obtained for the AC power flow study. Tables 4.5 and 4.6 present the real and imaginary components of the current in all loads, generators and branches. Table 4.7 detail the results of the loss allocation process to loads and Figure 4.3 illustrates the percentages of loss allocation to each demand bus.

Table 4.3 – Specified values and results from the AC power flow study.

Bus		Voltage		Generation		Load	
Node	Type	Module [pu]	Phase [rad]	MW	MVAr	MW	MVAr
1	Ref	1,001316602	0	0,097365134	0,062276238	0	0
2	PQ	0,995144674	-0,0097713	0	0	0	0
3	PQ	0,987956523	-0,0077184	0	0	0	0
4	PQ	0,981397508	-0,0063175	0	0	0,015	0,0036
5	PQ	0,975625543	-0,0034257	0	0	0	0
6	PV	0,98182293	0,0031481	0,025	-0,022796292	0	0
7	PQ	0,955331893	-0,0009994	0	0	0	0
8	PV	0,95	-0,0001537	0,016546952	0,005897566	0,072	0,024
9	PQ	0,968953521	-0,0032252	0	0	0	0
10	PV	0,968180157	-0,0010155	0,025	-0,003538894	0,03	0,016
11	PQ	0,963663469	-0,0054265	0	0	0	0
12	P	0,960889782	-0,0015549	0,003	-0,005273854	0,009	0,00330
13	PQ	0,95948383	-0,0085646	0	0	0	0
14	PV	0,95	-0,012837	0,011420244	0,024859088	0,047	0,012

Table 4.4 – Branch results from the power flow study.

Branch		Emission		Reception		Losses	
Node i	Node j	MW	MVAr	MW	MVAr	MW	MVAr
1	2	0,097365134	0,06227624	-0,097365134	-0,060943918	0	0,001332319
2	3	0,097365134	0,06094392	-0,096537431	-0,060702019	0,0008277	0,000241899
3	4	0,015105312	0,0036029	-0,015	-0,0036	0,00010531	2,90303E-06
3	5	0,08143212	0,05709912	-0,080172954	-0,056731121	0,00125917	0,000367996
5	6	-0,024692748	0,02281522	0,025	-0,022796292	0,00030725	1,89249E-05
5	7	0,057009377	0,0184043	-0,055779654	-0,018156869	0,00122972	0,000247429
7	8	0,055779654	0,01815687	-0,055453048	-0,018102434	0,00032661	5,44344E-05
5	9	0,047856328	0,01551161	-0,047525962	-0,015415057	0,00033037	9,65504E-05
9	10	0,005047191	0,0195434	-0,005	-0,019538894	4,7192E-05	4,50218E-06
9	11	0,042478775	-0,0041283	-0,042237719	0,004198787	0,00024106	7,04493E-05
11	12	0,006050571	0,00857744	-0,005999998	-0,008576044	5,0574E-05	1,39411E-06
11	13	0,036187153	-0,0127762	-0,035990105	0,012833813	0,00019705	5,7588E-05
13	14	0,035990106	-0,0128338	-0,035579755	0,012859088	0,00041035	2,52753E-05

Table 4.5 – Real and Imaginary components of the currents in loads and generations.

	Loads		Generations	
Node	Ireal (pu)	Iimag (pu)	Ireal (pu)	Iimag (pu)
1	0	0	0,243092779	0,15548588
2	0	0	0	0
3	0	0	0	0
4	0,0381521	0,009411808	0	0
5	0	0	0	0
6	0	0	0,06347405	-0,05824594
7	0	0	0	0
8	0,189464	0,063187016	0,043542225	0,0155266
9	0	0	0	0
10	0,0774229	0,041393269	0,064563348	-0,00907245
11	0	0	0	0
12	0,0234024	0,008622191	0,007826591	-0,01370912
13	0	0	0	0
14	0,1232687	0,033164036	0,029211041	0,06579905

Table 4.6.- Values of the real and imaginary components of the branch currents.

Node j	Node i	Ireal (pu)	Iimag (pu)	I (pu)
1	2	0,24309	0,15549	0,288565
2	3	0,24309	0,15549	0,288565
3	4	0,03815	0,00941	0,039296
3	5	0,20494	0,14607	0,251671
5	6	-0,0635	0,05825	0,086148
5	7	0,14592	0,04766	0,153508
7	8	0,14592	0,04766	0,153508
5	9	0,12249	0,04017	0,128911
9	10	0,01286	0,05047	0,052078
9	11	0,10963	-0,010298	0,110116
11	12	0,01558	0,02234	0,027231
11	13	0,09406	-0,032635	0,099558
13	14	0,09406	-0,032635	0,099558

Table 4.7 – Loss allocation results to loads in kW.

			Loads														
Branch	Bus 1	Bus 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total
1	1	2															0,0000
2	2	3				0,0524		0,1089		0,4346		0,0567		0,0043		0,1707	0,8277
3	3	4				0,1053											0,1053
4	3	5						0,1966		0,7007		0,0983		0,0071		0,2565	1,2592
5	6	5						0,1405		0,1132		0,0009		0,0014		0,0512	0,3073
6	5	7								1,2297							1,2297
7	7	8								0,3266							0,3266
8	5	9										0,0395		0,0111		0,2798	0,3304
9	9	10										0,0472					0,0472
10	9	11										0,0021		0,0064		0,2326	0,2411
11	11	12												0,0506			0,0506
12	11	13										0,0037		0,0175		0,1759	0,1970
13	13	14										0,0077		0,0364		0,3663	0,4104
Losses allocated to consumers (kW)			0	0	0	0,1577	0	0,4460	0	2,8048	0	0,2562	0	0,1347	0	1,5329	5,3324

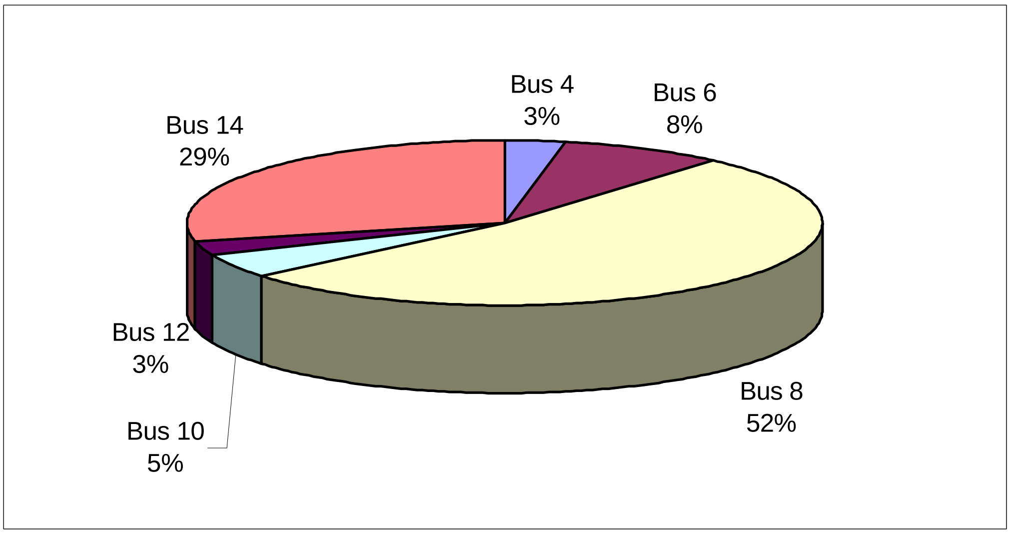

Figure 4.3 – Percentages of losses allocated to each demand bus.

4.3. Case 2

Case 2 differs from Case 1 since the loads in buses 10 and 12 increased from 30 to 50 kW and from 9 to 15 kW. Apart from that, the injected power from the main MV network increased from 97 kW to 107 kW, the power generated in bus 8 increased from 16 kW to 37 kW, the power generated in bus 10 decreased from 25 kW to 0, the power generated in bus 12 increased from 3 kW to 5 kW and the power generated in bus 14 increased from 11 to 31 kW. Total active losses increased from 5,33 kW in Case 1 to 7,51 kW in Case 2.

For Case 2, Tables 4.8 and 4.9 present the specified values and the results obtained for the AC power flow study. Tables 4.10 and 4.11 present the real and imaginary components of the current in all loads, generators and branches.

Table 4.8 – Specified values and results from the AC power flow study.

Bus		Voltage		Generation		Load	
Node	Type	Module [pu]	Phase [rad]	MW	MVAr	MW	MVAr
1	Ref	0,9994	0,0000	0,107669996	0,086365583	0	0
2	PQ	0,9908	-0,0109	0	0	0	0
3	PQ	0,9825	-0,0075	0	0	0	0
4	PQ	0,9760	-0,0061	0	0	0,015	0,0036
5	PQ	0,9680	-0,0005	0	0	0	0
6	PV	0,9735	0,0173	0,025	-0,063308322	0	0
7	PQ	0,9537	0,0088	0	0	0	0
8	PV	0,9500	0,0114	0,037060742	-0,009132645	0,072	0,024
9	PQ	0,9588	-0,0057	0	0	0	0
10	PV	0,9530	-0,0051	1,73472E-17	0,006040741	0,05	0,016
11	PQ	0,9564	-0,0104	0	0	0	0
12	PV	0,9522	-0,0257	0,005	0,036179743	0,015	0,00381
13	PQ	0,9542	-0,0103	0	0	0	0
14	PV	0,9500	-0,0091	0,031774023	0,006719891	0,047	0,012

Table 4.9 – Branch results from the power flow study.

Branch		Emission		Reception		Losses	
Node i	Node j	MW	MVAr	MW	MVAr	MW	MVAr
1	2	0,1077	0,0864	-0,1077	-0,0845	0,0000	0,0019
2	3	0,1077	0,0845	-0,1065	-0,0841	0,0012	0,0003
3	4	0,0151	0,0036	-0,0150	-0,0036	0,0001	0,0000
3	5	0,0914	0,0805	-0,0895	-0,0800	0,0019	0,0006
5	6	-0,0237	0,0634	0,0250	-0,0633	0,0013	0,0001
5	7	0,0360	0,0333	-0,0352	-0,0332	0,0008	0,0002
7	8	0,0352	0,0332	-0,0349	-0,0331	0,0002	0,0000
5	9	0,0772	-0,0168	-0,0764	0,0170	0,0008	0,0002
9	10	0,0503	0,0100	-0,0500	-0,0100	0,0003	0,0000
9	11	0,0261	-0,0270	-0,0259	0,0271	0,0002	0,0001
11	12	0,0105	-0,0324	-0,0100	0,0324	0,0005	0,0000
11	13	0,0153	0,0053	-0,0153	-0,0053	0,0000	0,0000
13	14	0,0153	0,0053	-0,0152	-0,0053	0,0001	0,0000

Table 4.10 – Real and Imaginary components of the currents in loads and generations.

	Loads		Generators	
Node	Ireal (pu)	Iimag (pu)	Ireal (pu)	Iimag (pu)
1	0	0	0,269336592	0,21604358
2	0	0	0	0
3	0	0	0	0
4	0,0383652	0,009455513	0	0
5	0	0	0	0
6	0	0	0,061379249	-0,16366545
7	0	0	0	0
8	0,1901814	0,060993838	0,097247958	-0,02514351
9	0	0	0	0
10	0,130949	0,042641109	0	0,01584644
11	0	0	0	0
12	0,0391124	0,011011866	0,010682188	0,09529584
13	0	0	0	0
14	0,1233917	0,032703151	0,083451466	0,01844409

Table 4.11 - Values of the real and imaginary components of the branch currents.

Node i	Node i	Ireal (pu)	Iimag (pu)	I (pu)
1	2	0,26941	0,21613	0,345391
2	3	0,26941	0,21616	0,345409
3	4	0,03835	0,00945	0,039499
3	5	0,23103	0,20657	0,309913
5	6	-0,0613	0,16371	0,174806
5	7	0,09293	0,08605	0,126652
7	8	0,09303	0,08621	0,12684
5	9	0,1994	-0,043289	0,204047
9	10	0,131	0,02682	0,13372
9	11	0,06845	-0,070011	0,097916
11	12	0,02833	-0,084403	0,089029
11	13	0,03985	0,01427	0,042325
13	14	0,03994	0,0143	0,042423

Table 4.12 – Loss allocation results to loads in kW.

			Loads														
Branch	Bus 1	Bus 2	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Total (kW)
1	1	2															0,0000
2	2	3				0,0645		0,3581		0,3223		0,3801		0,0190		0,0418	1,1859
3	3	4				0,1064											0,1064
4	3	5						0,6647		0,5300		0,6213		0,0293		0,0642	1,9094
5	6	5						1,1095		0,0508		0,0911		0,0043		0,0094	1,2651
6	5	7								0,8371							0,8371
7	7	8								0,2230							0,2230
8	5	9						0,0292		0,0081		0,6729		0,0379		0,0796	0,8277
9	9	10										0,3111					0,3111
10	9	11						0,0450		0,0118		0,0406		0,0313		0,0619	0,1906
11	11	12						0,1991		0,0557		0,1805		0,0547		0,0506	0,5406
12	11	13														0,0356	0,0356
13	13	14														0,0745	0,0745
Losses allocated to consumers (kW)			0	0	0	0,1709	0	2,4057	0	2,0387	0	2,2976	0	0,1765	0	0,4176	7,5070

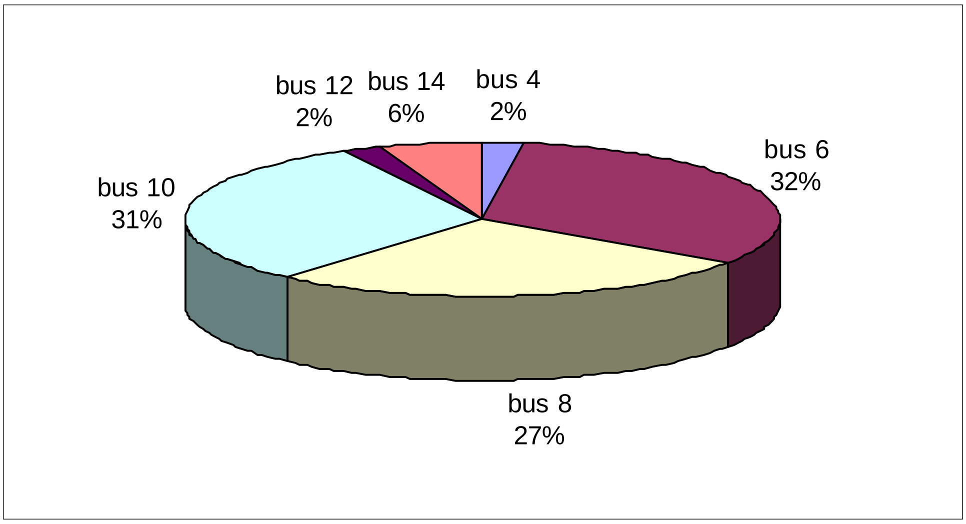

Figure 4.4 - Percentages of losses allocated to each demand bus.

4.4. Comments

The above simulations were performed assuming that generators are not charged for network losses from a regulatory point of view. This means that losses are of the responsibility of consumers and the amount collected this way can be distributed to microsources and to the main MV grid operator. This way, micro generators would have an extra revenue apart from the energy supplied to consumers thus contributing to improve their financial performance. In the above two simulations the losses should be paid by consumers in the percentages indicated in Figures 4.3 and 4.4.

The above simulations were performed considering extra reactive loads in buses 6, 10 and 12 in Case 1 and in buses 6 and 8 in Case 2. It should be noticed that there are already loads connected to buses 8, 10 and 12 so that, in fact, it was considered one only new load in bus 6. It was necessary to consider these loads since the microgenerators connected to these branches absorb reactive power, that is, they can be seen as reactive loads. Therefore, they should be allocated a part of active losses since reactive flows contribute to determine the branch current magnitudes that originate active losses. When addressing the buses 8, 10 and 12 it should be noticed that there are two reactive loads connected to each of them. One of those reactive loads is related to the consumer and the other derives from the reactive consumption of the microsource. If one wants to distribute the active losses by these two loads, we can adopt a simple proportional allocation, that is, allocate losses assigned to a bus in a proportional way regarding the loads connected to it. This problem does not exist for bus 6 since in this bus there is no consumer and the only reactive load is related with the microgenerator.

The percentages of losses allocated to each consumer vary when going from Case 1 to Case 2 as can be seen in Table 4.13. The patterns of loss allocations are very different mainly because the microsource connected to bus 6 increases the reactive power it is absorbing when going from Case 1 to Case 2 and because the load connected to bus 10 increased from 30 to 50 kW.

In Case 1 the microgenerator in bus 6 is absorbing 22,8 kVAr while in Case 2 this is increased to 63,3 kVAr. Since negative reactive generations are treated as loads, the branch flow pattern changes, namely when considering the reactive part, and so the branch current and losses also change. This ultimately leads to an increase of the percentage of active losses allocated to consumer 6 (in fact, the microgenerator in bus 6) from 8% to 32 %. The increase of the load in bus 10 explains the increase of the percentage of active losses allocated to this consumer (from 5 to 31%).

Table 4.13 – Comparison of the loss allocation in the two Cases.

	Case 1 (%)	Case 2 (%)
Consumer 4	3	2
Consumer 6	8	32
Consumer 8	52	27
Consumer 10	5	31
Consumer 12	3	2
Consumer 14	29	6

The above reasoning also means that these results can be used to send economic signals to network users:

- loads – they should choose their connection point in order to have a small percentage of allocated active losses to pay;
- generators - they should choose adequate connection nodes in order to avoid as much as possible the increase of the absorbed reactive power and thus the active losses to be allocated to them.

A different regulatory approach can also be adopted. In this second hypothesis, one should compute the losses and allocate them to the loads in the absence of the microsources. In a second phase, one should evaluate losses considering the microsources namely to compute how these microsources contribute to increase or decrease them. The avoided losses or the loss increases should then be allocated to the microsources leading to an extra revenue or to a payment. This way it would also be sent an economic signal to the microsources in order to select adequately their connection point.

These two approaches differ in the sense that the first one considers the microsources connected to the grid from the beginning. This means they are part of the grid so that it is not correct to evaluate scenarios in which they are not present. The second one is based on the comparison of two situations. This second approach seems not so adequate namely given the definition of a microgrid, that is, an association of a LV grid, loads and microsources having an high degree of controllable devices.

5. Final remarks

In this report, we presented a review of active loss allocation methods enumerating their advantages and drawbacks given a set of principles detailed in section 2.1. The methods that were analysed include the proportional allocation, marginal allocation, proportional sharing allocation, allocation using the impedance matrix, incremental allocation, allocation based on the results of OPF studies and approaches developed to allocate active losses to transactions.

In section 3, we described the loss allocation approach that was implemented. This approach is based in the proportional sharing principle according to which the flows entering any node are distributed proportionally between the outflows. The developed approach is structured in three phases. The first one aims at allocating active losses to consumers in the absence of micro generators, the second one considers that microgenerators are already connected to the grid and aims at evaluating and allocating the variations of losses and the third one evaluates and allocates the voltage related loss variations.

In section 4 we applied the developed loss allocation approach to a LV grid having a number of microsource connected to it. The algorithm was applied in two situations that differ because two consumptions were increased and the injected power from the MV grid increased by 10%. This lead to the increase of active losses by 41.5%. Another interesting aspect of this application comes from the fact that there are reactive power absorbed by some microsources. This is more evident in node 6. In this node the reactive absorption increases from 22,8 kVAr to 63,3 kVAr, meaning that the microsource connected to this node must be assigned an higher share of the active losses in the grid.

The above issues justify that the developed approach can be applied to distribution networks allocating in a successively way active losses to consumers (either generators absorbing reactive power or consumers) transmitting economic signals to grid users in order to induce more adequate connection points and more efficient uses of the grid.

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